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# A TWO-STAGE DATA ENVELOPMENT ANALYSIS MODEL WITH FEEDBACK AND FIXED-SUM UNDESIRABLE OUTPUTS: AN EMPIRICAL STUDY OF PROVINCIAL CARBON EMISSIONS IN CHINA

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## Abstract

Data Envelopment Analysis (DEA) is a pivotal method for evaluating the efficiency of multi-input and multi-output systems. The traditional DEA assumes that the output levels of all decision making units (DMUs) can be adjusted freely. However, in practice, there often exists some constraints such as feedback mechanisms and fixed-sum undesirable outputs, rendering the adjustment of output levels among DMUs no longer satisfying the assumption of independence. Given this, it is necessary to construct a two-stage DEA model with feedback and fixed-sum undesirable outputs, along with its substage efficiency decomposition model. The Generalized Equilibrium Efficient Frontier DEA (GEEFDEA) method is employed for efficiency evaluation. Furthermore, the proposed method is applied to assess provincial carbon emission efficiency in China, demonstrating its validity and practicality. This research reveals that: (1) Compared to conventional models, the GEEFDEA approach with feedback and fixed-sum undesirable outputs significantly enhances carbon emission efficiency. Especially, regions with high carbon efficiency are mainly located in the eastern and western of China. (2) The substage efficiency decomposition model answers the question of how carbon emission credits should be adjusted at each stage. In the energy production stage, provinces requiring increased emissions are mainly developed eastern regions and underdeveloped western areas, while those needing reductions are predominantly heavy industrial or resource-dependent provinces. In the energy utilization stage, the provinces that need to increase carbon emissions are mainly in the economically active regions, and those that need to reduce carbon emissions are mainly in the provinces with high-emission industries. This research provides critical decision-making insights for enhancing carbon emission efficiency across China’s regions.

## Keywords

Output Feedback; Non-Expected Output Fixed Sum; Data Envelopment Analysis; Carbon Emission; GEEFDEA

## 1. Introduction

The Data Envelopment Analysis (DEA) method proposed by Charnes et al. (1978) is a non-parametric approach for efficiency evaluation. As a data-driven frontier analysis technique, DEA's prominent advantage lies in its ability to operate without requiring prior assumptions about the potential functional relationships between inputs and outputs (Seiford and Thrall, 1990). Each decision-making unit (DMU) is treated as a black box, where its efficiency is measured by the ratio of its actual input-output levels to the optimal input-output levels. The traditional data enveloping analysis method assumes that the output level of all DMUs can be adjusted freely. However, in practice, there often exists some constraints such as feedback mechanisms and fixed-sum undesirable outputs, rendering the adjustment of output levels among DMUs no longer satisfying the assumption of independence. In this case, the output levels of decision-making units (DMUs) may influence each other, thereby affecting efficiency measurements. To tackle this, scholars proposed the concept of network DEA (Färe and Grosskopf, 1996), which can evaluate the efficiency of multi-input, multi-output systems. This approach treats a system as being composed of subsystem to reveal its internal structure and operational mechanisms. The two-stage DEA model is one type of network DEA models.

The two-stage DEA can be divided into single type and mixed type. The single type includes sequence type, feedback type, resource constraint type and fixed-sum undesirable outputs. The mixed type mainly refers to the two-stage DEA structure model with resource constraint and fixed-sum outputs. In particular, this has attracted extensive scholarly attention to the problem of assessing the efficiency of decision-making units with fixed-sum outputs. The fixed-sum outputs refers to the total output of the two substages is fixed. According to whether the output is expected output, it can be divided into fixed-sum expected outputs and fixed-sum unexpected output. Generally, typical expected outputs include total market share, total number of competition medals, etc (Li et al.,2023; Yang et al.,2014). Yang et al. (2014) adopt an equilibrium efficiency frontier data envelopment analysis approach for evaluating decision-making units with fixed-sum outputs. In general, unexpected output refers to carbon emissions (Gomes and Lins, 2008). In particular, with the rapid economic and social development, the issue of carbon emissions has become crucial to the realization of global sustainable development. In response, China has proposed a goal of dual-carbon, which can be achieved by reducing energy consumption. At the same time, China has set targets for reducing energy consumption in its 12th Five-Year plans, reducing energy consumption per unit of GDP by 16%. The prominent feature of this policy is that the authorities limit carbon emissions to a specific level, and then they restrict the relevant production behaviors of enterprises and even the entire industry, so as to achieve green and sustainable development of the economy. Correspondingly, Gomes and Lins ( 2008) extended the DEA model to the case of fixed-sum unexpected output. Li et al. (2021) established a generalized equilibrium effective frontier DEA model based on the case of fixed-sum outputs, and evaluated the carbon emission performance of 30 provincial regions in China.

As a matter of fact, the problem of energy efficiency assessment is not only a problem with fixed-sum unexpected output, but also contains feedback characteristics. Specifically, energy efficiency assessment consists of two main phases, namely the energy production and energy utilization phases. In the energy production stage, we takes labor, capital and energy consumption as inputs, and takes energy generation and carbon emissions as outputs. And in the energy utilization stage, we takes the energy generation in the first stage as inputs, and takes GDP and carbon emissions as outputs. Meanwhile, the GDP generated in the energy utilization stage is fed back to the energy production stage as an input factor. However, existing studies have not investigated the two-stage dea model with fixed-sum unexpected output and feedback characteristics. Given this, it is necessary to construct a two-stage DEA model with feedback and fixed-sum undesirable outputs, along with its substage efficiency decomposition model. The Generalized Equilibrium Efficient Frontier DEA (GEEFDEA) method is employed for efficiency evaluation. Furthermore, the proposed method is applied to assess provincial carbon emission efficiency in China, demonstrating its validity and practicality.

This research reveals that: (1) Compared to conventional models, the GEEFDEA approach with feedback and fixed-sum undesirable outputs significantly enhances carbon emission efficiency. Especially, regions with high carbon efficiency are mainly located in the eastern and western of China. (2) The substage efficiency decomposition model answers the question of how carbon emission credits should be adjusted at each stage. In the energy production stage, provinces requiring increased emissions are mainly developed eastern regions and underdeveloped western areas, while those needing reductions are predominantly heavy industrial or resource-dependent provinces. In the energy utilization stage, the provinces that need to increase carbon emissions are mainly in the economically active regions, and those that need to reduce carbon emissions are mainly in the provinces with high-emission industries. This research provides critical decision-making insights for enhancing carbon emission efficiency across China's regions.

The research contributions of this paper are as follows: firstly, we expand the two-stage DEA network structure model to a new situation, which considers both feedback and fixed-sum undesirable outputs, filling the current research gap. Secondly, we use GEEFDEA method to the constructe the equilibrium efficient frontier ,which answer the question of how to improve the carbon emission efficiency of 30 provincial-level regions in China. Thirdly,we also analyze the carbon emission efficiency of eastern, central, western and northeastern regions of China, so as we can put forward feasible policy suggestions to improve the carbon emission efficiency of China.

## 2.Literature Review

### *2.1 Types of two-stage DEA*

In existing literatures, the two-stage DEA have been applied to various fields, such as sports, banking, enterprises, environment, etc. As shown in table 1, the types of two-stage DEA can be divided into single type and mixed type. The single type includes sequence type, feedback type, resource constraint type and fixed-sum undesirable outputs. The mixed type mainly refers to the two-stage DEA structure model with resource constraint and fixed-sum outputs. The sequential two-stage DEA means that all the output in the first stage is input in the second stage. Wang et al. (1997) studied the impact of the utilization efficiency of information technology on bank performance, and took all the deposits generated in the first stage as input in the second stage of profit process. Kao and Hwang(2008) analyzed that non-life insurance companies used all the premiums generated in the first stage as inputs for the second stage of underwriting and investment profit process. The feedback two-stage DEA means that part of the output in the second stage becomes the input in the first stage. For example, Liang et al. (2011) studied the performance of 50 universities in China, taking the research funds generated in the second stage as the input in the first stage.The resource-constrained two-stage DEA refers to the shared input of the two sub-stages. For example, Bi et al. (2009) used a resource-constrained two-stage DEA efficiency evaluation model to evaluate the efficiency of a state-owned commercial bank in a China. The fixed-sum outputs refers to the total output of the two substages is fixed. According to whether the output is expected output, it can be divided into fixed-sum expected outputs and fixed-sum unexpected output. Generally, typical expected outputs include total market share, total number of competition medals, etc. (Li et al.,2023; Yang et al.,2014). However, unexpected output refers to carbon emissions (Gomes and Lins, 2008). For example, Li et al. (2021) established a generalized equilibrium effective frontier DEA model based on the case of fixed-sum outputs, and evaluated the carbon emission performance of 30 provincial regions in China. Of course, some scholars also considered a mixed two-stage DEA model. For example, Li et al. (2016) researched the network structure of two-stage DEA with sharing input and sharing output, and took 17 branches of China Construction Bank in Anhui Province as an example for verification and analysis. However, there are few studies on the application of two-stage DEA model with feedback and fixed-sum undesirable outputs.

**Table 1 Classification of two-stage DEA models**

| Type | Characteristic | Applications | Literature |
|---|---|---|---|
| Sequence | All the output in the first stage is input in the second stage | Financial intermediation | Wang et al. 1997); Kao and Hwang 2008) |
| Feedback | Part of the output in the second stage becomes the input in the first stage | Research funds | Liang et al. (2011) |
| Resource constraint | Shared input of the two sub-stage | Bank deposits and loans | Bi et al. (2009) |
| Fixed-sum outputs | The total output of the two substages is fixed. | Total market share, total number of competition medals, carbon emissions | Li et al.,(2023), Yang et al.(2014), Gomes and Lins( 2008) |
| Mixed | The network structure of two-stage DEA with sharing input and sharing output | Bank operational efficiency | Li et al. (2016) |

### *2.2 Two-Stage DEA Efficiency Evaluation Methods*

Authorities limit carbon emissions to a specific level and any excessive emission behavior is not allowed (Li et al.,2020; Zhu et al.,2020). Based on this, two-stage DEA efficiency evaluation methods with fixed-sum outputs are particularly important. For example,Li et al.(2022) measured the energy production and utilization efficiency of thermal power industry under carbon emission and fixed scenarios. Zha et al.(2016) proposed a radial stochastic DEA model based on opportunity constrained planning. Then, the model was used to measure energy utilization and $CO_2$ emission efficiencies in China. However, Chen and Zhu (2004) argued that the traditional model could not be effectively applied to the two-stage dea model and proposed a two-stage production model connected by intermediate outputs. Moreover, most of the traditional dea models are radial measurement models, which require all inputs or outputs to be improved in the same proportion. It limits the room for improvement of inputs and outputs, making the measurement results appear weak DEA effective and leading to incomparable efficiency results for more DMUs. In this regard, scholars have proposed non-radial measurement models that allow different input-output indicators to be improved at different ratios, which increases the room for improvement. Kao and Hwang (2008) proposed that the overall efficiency is the product of the efficiencies of the two sub-processes by considering the crosstalk between the two sub-stages. Chen et al. (2009), on the other hand, argued that the overall efficiency should be expressed as a weighted sum of the two sub-process efficiencies. The method is able to solve the efficiency of a two-stage dea model under the assumption of constant returns to scale (CRS) or variable returns to scale (VRS). For example, Chen et al. (2010) established the a two-stage dea model for evaluating shared inputs. The model was subsequently extended to the case where intermediate outputs are only partially but not fully consumed in the second stage, where some of the outputs from the first stage are used as final outputs and where the second stage has independent non-negative inputs (Ma, 2015; Izadikhah et al., 2018).

The methods for evaluating two-stage DEA models of fixed sums can be divided into fixed-sum outputs DEA (FSODEA) approach (Yang et al., 2011),equilibrium efficient frontier data envelopment analysis (EEFDEA) approach (Yang et al., 2014) and generalized equilibrium efficient frontier data envelopment analysis (GEEFDEA) approach. Among them, FSODEA is also known as the minimum reduction strategy, which means that all decision units make the least adjustment to construct a new efficient boundary. However, the FSODEA model makes it possible for DMUs to be projected on different efficient boundaries, making DMUs not comparable. In this regard, Yang et al. (2014) proposed the equilibrium efficient frontier data envelopment analysis approach. Compared to FSODEA approach, this approach makes all DMUs based on the same equilibrium efficient frontier. However, this approach has three drawbacks. First, it is necessary to determine the evaluation order in advance, which is subjective. In other words, different decision order will lead to different evaluation results. Second, the equilibrium effective frontier can be achieved step by step, which increases the calculation burden when the number of DMUs is large. Third, the signs of all outputs' adjustments must be the same for each DMU. For the above reasons, Yang et al. (2015)proposed a generalized equilibrium efficient frontier data envelopment analysis approach which improves and strengthens the EEFDEA approach. Compared to EEFDEA approach, this approach not only maintains all advantages of EEFDEA approach, but have some advantages. To be specific, there is no need to take the decision order of DMUs into consideration, and the equilibrium efficient frontier can be reached in just one step. And it is also not necessary that the the signs of all outputs' adjustments must be the same for each DMU. Therefore, the GEEFDEA has significant advantages and good applicability to the two-stage data envelopment analysis model with fixed-sum undesirable outputs. It provides a good reference for this study.

### *2.3 Expression of unexpected outputs*

The mathematical treatment of unexpected outputs is shown in Table 2. It includes treating unexpected outputs as inputs(Hailu and Veeman,2001), data transformation methods (Seiford and Zhu, 2002), hyperbolic modeling(Färe et al.,2024), directional distance function (Li et al., 2020) and ecological inefficiency methods (Chen and Delmas, 2012). The method that considers unexpected outputs as inputs reduces unexpected outputs by minimizing inputs. The method need not to change the framework of DEA. It has the significant advantage of being simple and easy. However, the method does not correspond to the reality of production. The data transformation method involves taking the inverse, the logarithm, and other mathematical function forms to transform them into positive outputs. For example, multiply the unexpected outputs by negative one and add an appropriate positive, making it positive. This method is compatible with traditional DEA models and can better reflect the impact of unexpected output on overall efficiency. However, it could distort the efficiency frontier if not handled properly. The hyperbolic model allows unexpected outputs to be adjusted in synchronization with expected outputs in hyperbolic proportions. It is able to maintain the technical substitution relationship between outputs. However, the method has complexity in nonlinear solution. The directional distance function method customizes the optimization path through the directional vector. It can also turn unexpected outputs into components of a new productivity index. However, direction vector selection is subjective. The ecological inefficiency method combines unexpected outputs with economic benefits into an efficiency ratio. The method is able to visualize the trade-off between economic and environment, but the setting of the ratio is subjective.

In summary, each method has its own advantages and disadvantages, which can be chosen according to the actual situation. Based on the model characteristics, this paper adopts the data transformation method to deal with the unexpected outputs.

**Table 2 Expression of unexpected outputs**

| Type | Explanation | Advantages | Disadvantages |
|---|---|---|---|
| As inputs | It reduces unexpected outputs by minimizing inputs | Simple and easy | Not correspond to the reality of production |
| Data transformation methods | It transform unexpected outputs into positive outputs | It is compatible with traditional DEA models | It could distort the efficiency frontier |
| Hyperbolic modeling | It can be adjusted in synchronization with expected outputs | It is able to maintain the technical substitution relationship between outputs | Complexity |
| Directional distance function | It customizes the optimization path through the directional vector | It can be turned into components of a new productivity index | The direction vector selection is subjective |
| Ecological inefficiency methods | It combines unexpected outputs with economic benefits | Reflect the trade-off between economic and environment | The ratio is subjective. |

## 3.Models

### *3.1 Traditional model*

Consider that we have *n* DMUs, denoted as $DMU_j\,(j=1,2,...,n)$. As shown in Figure 1, in the first stage, each of DMUs consumes input $X_{ij}\,(i=1,2,...,m)$ to produce expected intermediate output $Z_{pj}\,(p=1,2,...,q)$ and unexpected intermediate output $Y_{cj}^1(c=1,2,...,d)$. In the second stage, the intermediate output both $Z_{pj}\,(p=1,2,...,q)$ and $Y_{cj}^1(c=1,2,...,d)$ in the first stage are used as inputs to obtain the expected final output $W_{gj}\,(g=1,2,...,r)$ and unexpected final output $Y_{cj}^2(c=1,2,...,d)$ It should be noted that the sum of unexpected output $Y_{cj}^1$ and $Y_{cj}^2$ is fixed, and it can be expressed as $Y_{cj}^1+Y_{cj}^2=Y_{cj}\,(c=1,2,...,d)$.

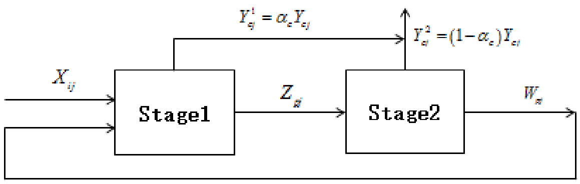



**Fig. 1 Two-stage DEA model with feedback and fixed-sum undesirable outputs**

Referring to the research of Chen et al.(2010), we adopt the weighted summation method to evaluate the efficiency of a two-stage system with feedback and fixed-sum undesirable outputs. As for the feedback output, we refer to the research of Liang et al.(2011) and believe that feedback variables play a dual role, that is, it is not only the final output of the second stage but also the input of the first stage.

Based on the feasibility of this research, we adopt the data conversion method to deal with the undesired output. Specifically, the undesired output was multiplied by negative 1, and then it was added an appropriate forward conversion variable to become positive. That is, suppose the actual value of the undesired output is $U_{cj}$, then $Max_j U_{cj}$ represents the maximum undesired output of the DMUs. Thus, the undesired output is converted to $Y_{cj}=Max_j U_{cj}+1-U_{cj}$. Therefore, we obtain the BCC model of two-stage efficiency evaluation, as shown in formula (1). As we can see, $v_i$、$\lambda_p$、$\delta_c$、$\beta_g$ represent the non-negative weights of initial input, intermediate output, undesired output and feedback variables respectively. We notice that $u_1$、$u_2$ are free variables in two substages respectively. When $u_1=u_2=0$, the BCC model will degenerate into a CCR model. $E_{1j}$、$E_{2j}$ represent the relative efficiency of the two sub-stages respectively, expressed by the ratio of input to output at each stage., and there is $E_{1j}\le 1$、$E_{2j}\le 1$ $\varphi_1$、$\varphi_2$ represents the weight of two sub-stage efficiencies in the overall efficiency,and there is $\varphi_1+\varphi_2=1$.

$$\text{Max } \mathrm{E}_0 = \varphi_1 \mathrm{E}_{10} + \varphi_2 \mathrm{E}_{20}$$
$$s.t.\begin{cases} \mathrm{E}_{1j} = \dfrac{\sum_{p=1}^{q}\lambda_p Z_{pj} + \sum_{c=1}^{d}\delta_c Y_{cj}^1 + u_1}{\sum_{i=1}^{m}\nu_i X_{ij} + \sum_{g=1}^{r}\beta_g W_{gj}} \le 1, \forall j \\ \mathrm{E}_{2j} = \dfrac{\sum_{c=1}^{d}\delta_c Y_{cj}^2 + \sum_{g=1}^{p}\beta_g W_{gj} + u_2}{\sum_{p=1}^{q}\lambda_p Z_{pj}} \le 1, \forall j \\ \varphi_1 + \varphi_2 = 1 \\ \lambda_p, \delta_c, \nu_i, \beta_g \ge 0, \forall p, c, i, g \end{cases} \tag{1}$$

Reference to the research of Chen et al. (2010), $\varphi_1$、$\varphi_2$ are represented as $\varphi_1 = \dfrac{\sum_{i=1}^{m}\nu_i X_{ij} + \sum_{g=1}^{r}\beta_g W_{gj}}{\sum_{i=1}^{m}\nu_i X_{ij} + \sum_{p=1}^{q}\lambda_p Z_{pj} + \sum_{g=1}^{r}\beta_g W_{gj}}$, $\varphi_2 = \dfrac{\sum_{p=1}^{q}\lambda_p Z_{pj}}{\sum_{i=1}^{m}\nu_i X_{ij} + \sum_{p=1}^{q}\lambda_p Z_{pj} + \sum_{g=1}^{r}\beta_g W_{gj}}$. There is a nonlinear fractional programming model , as shown in formula (2).

$$\text{Max } \mathrm{E}_0 = \left(\sum_{p=1}^{q}\lambda_p Z_{pj} + \sum_{c=1}^{n}\delta_c Y_{cj}^1 + u_1 + \sum_{c=1}^{n}\delta_c Y_{cj}^2 + \sum_{g=1}^{p}\beta_g W_{gj} + u_2\right) \Big/ \left(\sum_{i=1}^{m}\nu_i X_{ij} + \sum_{g=1}^{r}\beta_g W_{gj} + \sum_{p=1}^{q}\lambda_p Z_{pj}\right)$$
$$s.t.\begin{cases} \mathrm{E}_{1j} = \dfrac{\sum_{p=1}^{q}\lambda_p Z_{pj} + \sum_{c=1}^{d}\delta_c Y_{cj}^1 + u_1}{\sum_{i=1}^{m}\nu_i X_{ij} + \sum_{g=1}^{r}\beta_g W_{gj}} \le 1, \forall j \\ \mathrm{E}_{2j} = \dfrac{\sum_{c=1}^{d}\delta_c Y_{cj}^2 + \sum_{g=1}^{p}\beta_g W_{gj} + u_2}{\sum_{p=1}^{q}\lambda_p Z_{pj}} \le 1, \forall j \\ \lambda_p, \delta_c, \nu_i, \beta_g \ge 0, \forall p, c, i, g \end{cases} \tag{2}$$

Using the Charnes-Cooper transform, we let $t = \dfrac{1}{\sum_{i=1}^{m}\nu_i X_{ij} + \sum_{p=1}^{q}\lambda_p Z_{pj} + \sum_{g=1}^{r}\beta_g W_{gj}}$, $\lambda_p^{'} = t\lambda_p, \delta_c^{'} = t\delta_c, \nu_i^{'} = t\nu_i, \beta_g^{'} = t\beta_g, u_1^{'} = tu_1,, u_2^{'} = tu_2$， then model (2) can be transformed into a linear model (3).

$$\text{Max } \mathrm{E}_0 = \sum_{p=1}^{q}\lambda_p^{'} Z_{pj} + \sum_{c=1}^{d}\delta_c^{'} Y_{cj}^1 + u_1^{'} + \sum_{c=1}^{d}\delta_c^{'} Y_{cj}^2 + \sum_{g=1}^{p}\beta_g^{'} W_{gj} + u_2^{'}$$
$$s.t.\begin{cases} \sum_{i=1}^{m}\nu_i^{'} X_{ij} + \sum_{g=1}^{r}\beta_g^{'} W_{gj} + \sum_{p=1}^{q}\lambda_p^{'} Z_{pj} = 1 \\ \sum_{p=1}^{q}\lambda_p^{'} Z_{pj} + \sum_{c=1}^{d}\delta_c^{'} Y_{cj}^1 + u_1^{'} - \sum_{i=1}^{m}\nu_i^{'} X_{ij} - \sum_{g=1}^{r}\beta_g^{'} W_{gj} \le 0, \forall j \\ \sum_{c=1}^{d}\delta_c^{'} Y_{cj}^2 + \sum_{g=1}^{p}\beta_g^{'} W_{gj} + u_2^{'} - \sum_{p=1}^{q}\lambda_p^{'} Z_{pj} \le 0, \forall j \\ \lambda_p^{'}, \delta_c^{'}, \nu_i^{'}, \beta_g^{'} \ge 0, \forall p, c, i, g \end{cases} \tag{3}$$

In general, the optimal two-stage efficiency can be obtained by solving model (3). However, it is difficult to accurately measure the efficiency evaluation in two-stage DEA with feedback and fixed-sum undesirable outputs, so it is necessary to construct the general equilibrium effective frontier.

### *3.2 GEEFDEA model*

This paper constructs the equilibrium efficient frontier referring to Yang et al. (2014). That is, when there are constraints of DEA with feedback and fixed-sum undesirable outputs, minimize the weighted adjustments such that all decision units lie on the efficient frontier. Combining the objective function in the additive efficiency model (2) of the two-stage DEA, we can obtain model (4). $\alpha_c$ and $1\text{-}\alpha_c$ represent the proportion of undesired outputs to total outputs in phases I and II, respectively. The first constraint indicates that the two sub-phases are adjusted from $\alpha_c Y_{cj}$, $Y_{cj}-\alpha_c Y_{cj}$ to $\alpha_c Y_{cj}+\eta_{cj}^1$, $Y_{cj}-\alpha_c Y_{cj}+\eta_{cj}^2$ respectively, which can reach the equilibrium efficient frontier. The second constraint indicates that the sum of the adjustments of all decision units is 0. The third and fourth constraints indicate that only the non-negative adjustments to the decision unit are calculated. The fifth and sixth constraints indicate that the output must satisfy the non-negative requirement after being adjusted. In general, $\alpha_c$ is set within a certain range, $L_c \le \alpha_c \le H_c$. Its upper and lower bounds are set according to the application scenario, as shown in the seventh constraint.

$$\begin{aligned}
&\text{Min } \sum_{j=1}^{n}\sum_{c=1}^{d}\delta_c\left(\theta_{cj}^1+\theta_{cj}^2\right)\\
&s.t.\begin{cases}
\left(\sum_{p=1}^{q}\lambda_p Z_{pj}+\sum_{c=1}^{n}\delta_c(\alpha_c Y_{cj}+\eta_{cj}^1)+u_1+\sum_{c=1}^{n}\delta_c(Y_{cj}-\alpha_c Y_{cj}+\eta_{cj}^2)+\sum_{g=1}^{p}\beta_g W_{gj}+u_2\right)\Big/\left(\sum_{i=1}^{m}\nu_i X_{ij}+\sum_{g=1}^{r}\beta_g W_{gj}+\sum_{p=1}^{q}\lambda_p Z_{pj}\right)=1,\forall j\\
\sum_{j=1}^{n}\left(\eta_{cj}^1+\eta_{cj}^2\right)=0,\forall c\\
\theta_{cj}^1=\max\left\{0,\eta_{cj}^1\right\},\forall c,j\\
\theta_{cj}^2=\max\left\{0,\eta_{cj}^2\right\},\forall c,j\\
\alpha_c Y_{cj}+\eta_{cj}^1\ge 0,\forall c,j\\
Y_{cj}-\alpha_c Y_{cj}+\eta_{cj}^2\ge 0,\forall c,j\\
L_c\le\alpha_c\le H_c,\forall c\\
\lambda_p,\delta_c,\nu_i,\beta_g\ge 0,\forall p,c,i,g
\end{cases}
\end{aligned}\tag{4}$$

Model (5) can be obtained by model (4), which is transformed using the absolute value method, which is transformed using the absolute value method.

$$\begin{aligned}
&\text{Min } \frac{1}{2}\sum_{j=1}^{n}\sum_{c=1}^{d}\delta_c\left(\left|\eta_{cj}^1\right|+\left|\eta_{cj}^2\right|\right)\\
&s.t.\begin{cases}
\left(\sum_{p=1}^{q}\lambda_p Z_{pj}+\sum_{c=1}^{n}\delta_c(\alpha_c Y_{cj}+\eta_{cj}^1)+u_1+\sum_{c=1}^{n}\delta_c(Y_{cj}-\alpha_c Y_{cj}+\eta_{cj}^2)+\sum_{g=1}^{p}\beta_g W_{gj}+u_2\right)\Big/\left(\sum_{i=1}^{m}\nu_i X_{ij}+\sum_{g=1}^{r}\beta_g W_{gj}+\sum_{p=1}^{q}\lambda_p Z_{pj}\right)=1,\forall j\\
\sum_{j=1}^{n}\left(\eta_{cj}^1+\eta_{cj}^2\right)=0,\forall c\\
\alpha_c Y_{cj}+\eta_{cj}^1\ge 0,\forall c,j\\
Y_{cj}-\alpha_c Y_{cj}+\eta_{cj}^2\ge 0,\forall c,j\\
L_c\le\alpha_c\le H_c,\forall c\\
\lambda_p,\delta_c,\nu_i,\beta_g\ge 0,\forall p,c,i,g
\end{cases}
\end{aligned}\tag{5}$$

Let $\delta_c\left|\eta_{cj}^k\right|+\delta_c\eta_{cj}^k=\psi_{cj}^k,\delta_c\left|\eta_{cj}^k\right|-\delta_c\eta_{cj}^k=\tau_{cj}^k,k=1,2$, then $\delta_c\left|\eta_{cj}^k\right|=\frac{1}{2}\left(\psi_{cj}^k+\tau_{cj}^k\right)$, $\delta_c\eta_{cj}^k=\frac{1}{2}\left(\psi_{cj}^k-\tau_{cj}^k\right),\psi_{cj}^k,\tau_{cj}^k\ge 0$. Let $\phi_c=\delta_c\alpha_c$ and add the constraint $\sum_{i=1}^{m}\nu_i X_{ij}+\sum_{g=1}^{r}\beta_g W_{gj}+\sum_{p=1}^{q}\lambda_p Z_{pj}\ge K$, where $K$ is positive. This means that the first constraint in model (5) is not equal to 0. Thus, the model (6) is obtained, which can be solved for the optimal solution $\eta_{cj}^{1*}=\left(\psi_{cj}^{1*}-\tau_{cj}^{1*}\right)\Big/2\delta_c^*$ and $\eta_{cj}^{2*}=\left(\psi_{cj}^{2*}-\tau_{cj}^{2*}\right)\Big/2\delta_c^*$.

$$\begin{aligned}
&\text{Min } \frac{1}{4}\sum_{j=1}^{n}\sum_{c=1}^{d}\left(\psi_{cj}^1+\tau_{cj}^1+\psi_{cj}^2+\tau_{cj}^2\right)\\
&s.t.\begin{cases}
\sum_{p=1}^{q}\lambda_p Z_{pj}+\frac{1}{2}\sum_{c=1}^{n}(\psi_{cj}^1-\tau_{cj}^1)+u_1+\sum_{c=1}^{n}\delta_c Y_{cj}+\frac{1}{2}\sum_{c=1}^{n}(\psi_{cj}^2-\tau_{cj}^2)+\sum_{g=1}^{p}\beta_g W_{gj}+u_2\\
=\sum_{i=1}^{m}\nu_i X_{ij}+\sum_{g=1}^{r}\beta_g W_{gj}+\sum_{p=1}^{q}\lambda_p Z_{pj},\forall j\\
\sum_{i=1}^{m}\nu_i X_{ij}+\sum_{g=1}^{r}\beta_g W_{gj}+\sum_{p=1}^{q}\lambda_p Z_{pj}\ge K,\forall j\\
\sum_{j=1}^{n}\left(\psi_{cj}^1-\tau_{cj}^1+\psi_{cj}^2-\tau_{cj}^2\right)=0,\forall c\\
2\phi_c Y_{cj}+\psi_{cj}^1-\tau_{cj}^1\ge 0,\forall c,j\\
2\delta_c Y_{cj}-2\phi_c Y_{cj}+\psi_{cj}^2-\tau_{cj}^2\ge 0,\forall c,j\\
\delta_c L_c\le\phi_c\le\delta_c H_c,\forall c\\
\lambda_p,\delta_c,\nu_i,\beta_g,\psi_{cj}^1,\tau_{cj}^1,\psi_{cj}^2,\tau_{cj}^2\ge 0,\forall p,c,i,g
\end{cases}
\end{aligned}\tag{6}$$

After obtaining the optimal adjustments of all decision units $\eta_{cj}^{1*}, \eta_{cj}^{2*}$, the common equilibrium effective frontier can be obtained. That is to say, all the adjusted decision units are located on the effective frontier at the same time. Therefore, the relative efficiency of the evaluated decision units can be calculated, as shown in model (7).

$$\text{Max } E_0^* = \left(\sum_{p=1}^{q}\lambda_p Z_{p0} + \sum_{c=1}^{n}\delta_c\alpha_c Y_{c0} + u_1\right) \Bigg/ \left(\sum_{i=1}^{m}\nu_i X_{i0} + \sum_{g=1}^{r}\beta_g W_{g0} + \sum_{p=1}^{q}\lambda_p Z_{p0}\right)$$
$$+\left(\sum_{c=1}^{n}\delta_c\left(Y_{c0}\text{-}\alpha_c Y_{c0}\right) + \sum_{g=1}^{p}\beta_g W_{g0} + u_2\right) \Bigg/ \left(\sum_{i=1}^{m}\nu_i X_{i0} + \sum_{g=1}^{r}\beta_g W_{g0} + \sum_{p=1}^{q}\lambda_p Z_{p0}\right)$$
$$s.t.\begin{cases} \hat{E}_{1j} = \dfrac{\sum_{p=1}^{q}\lambda_p Z_{pj} + \sum_{c=1}^{d}\delta_c\left(\alpha_c Y_{cj} + \eta_{cj}^{1*}\right) + u_1}{\sum_{i=1}^{m}\nu_i X_{ij} + \sum_{g=1}^{r}\beta_g W_{gj}} \le 1, \forall j \\ \hat{E}_{2j} = \dfrac{\sum_{c=1}^{n}\delta_c\left(Y_{c0}\text{-}\alpha_c Y_{c0} + \eta_{cj}^{2*}\right) + \sum_{g=1}^{p}\beta_g W_{gj} + u_2}{\sum_{p=1}^{q}\lambda_p Z_{pj}} \le 1, \forall j \\ L_c \le \alpha_c \le H_c, \forall c \\ \lambda_p, \delta_c, \nu_i, \beta_g \ge 0, \forall p, c, i, g \end{cases} \tag{7}$$

When $t = \frac{1}{\sum_{i=1}^{m}\nu_i X_{ij} + \sum_{p=1}^{q}\lambda_p Z_{pj} + \sum_{g=1}^{r}\beta_g W_{gj}}$ and $\lambda_p^{'} = t\lambda_p, \delta_c^{'} = t\delta_c, \nu_i^{'} = t\nu_i, \beta_g^{'} = t\beta_g$, model (7) is transformed into a linear model (8) by CC transformation. The optimal objective function value can be obtained by solving model (8). That is, the overall efficiency of the two-stage DEA with with feedback and fixed-sum undesirable outputs can be obtained. When $E_0^* < 1, E_0^* = 1, E_0^* > 1$, it represents the case that the decision unit is ineffective, efficient and super efficient, respectively.

$$\text{Max } E_0^* = \sum_{p=1}^{q}\lambda_p^{'} Z_{p0} + \sum_{c=1}^{d}\delta_c^{'} Y_{c0} + u_1^{'} + \sum_{g=1}^{p}\beta_g^{'} W_{g0} + u_2^{'}$$
$$s.t.\begin{cases} \sum_{i=1}^{m}\nu_i^{'} X_{i0} + \sum_{g=1}^{r}\beta_g^{'} W_{g0} + \sum_{p=1}^{q}\lambda_p^{'} Z_{p0} = 1 \\ \sum_{p=1}^{q}\lambda_p^{'} Z_{pj} + \sum_{c=1}^{d}\phi_c^{'} Y_{cj} + \sum_{c=1}^{d}\delta_c^{'}\eta_{cj}^{1*} + u_1^{'} - \sum_{i=1}^{m}\nu_i^{'} X_{ij} - \sum_{g=1}^{r}\beta_g^{'} W_{gj} \le 0, \forall j \\ \sum_{g=1}^{p}\beta_g^{'} W_{gj} + \sum_{c=1}^{d}\delta_c^{'} Y_{c0} - \sum_{c=1}^{d}\phi_c^{'} Y_{cj} + \sum_{c=1}^{d}\delta_c^{'}\eta_{cj}^{2*} + u_2^{'} - \sum_{p=1}^{q}\lambda_p^{'} Z_{pj} \le 0, \forall j \\ \lambda_p^{'}, \delta_c^{'}, \nu_i^{'}, \beta_g^{'}, \alpha_c \ge 0, \forall p, c, i, g \end{cases} \tag{8}$$

## 4.Model Application

### *4.1 Model description and data sources*

This section evaluates and analyzes the two-stage DEA carbon emission efficiency assessment problem with with feedback and fixed-sum undesirable outputs that includes energy production and energy use. Based on the $C-D$ production function, the model of $F(GDP, CO_2) = f(K, L, E)$ is constructed, which uses capital $K$, labor $L$ and energy $E$ as inputs, $GDP$ as desired output, and $CO_2$ as undesired output. At the energy utilization stage, the intermediate variable $EO$ is used as an input, $GDP$ and $CO_2$ are as outputs. The dual variable $GDP$, is not only the output of the energy utilization stage, but also feeds back into the energy production stage as a input. Capital is calculated using the perpetual inventory method proposed by Zhang et al. (2004). Labor is expressed as the number of people employed in each region. Energy consumption and energy output are calculated using standard coal. Carbon emissions were calculated according to the carbon footprint formula of the 2006 IPCC Guidelines for National Greenhouse Gas Inventories (Zhu et al., 2020).

All statistical data in this paper are mainly obtained from China Statistical Yearbook, China Environmental Statistical Yearbook, China Energy Statistical Yearbook and statistical yearbooks of provinces and municipalities. The missing individual data are supplemented by interpolation. In this paper, the data at the provincial level of 30 provinces and autonomous regions in China in 2022 are selected for analysis. Considering the availability of data from the Tibet Autonomous Region (TAR), Hong Kong, Macao, and Taiwan, they are not included in the study sample.

### *4.2 Results analysis*

#### 4.2.1 Descriptive statistical analysis

As shown in Table 3, descriptive statistics of the indicators related to energy production and utilization processes in 30 provincial regions of China in 2022 are listed, including the mean, standard deviation, minimum and maximum values. The mean value of capital is 75140.06, with minimum and maximum values of 12682.15 and 191089.90 respectively. The mean value of labor is 759.67, with minimum and maximum values of 42.87 and 3501.57 respectively. The mean value of energy consumption is 38906.79, with minimum and maximum values of 5447.70 and 110553.60 respectively. The mean value of dual variable energy output is 16372.59, with minimum and maximum values of 701.13 and 113160.60 respectively. The mean value of desired output gross regional product is 30524.59, with minimum and maximum values of 2766.74 and 100815.60 respectively. The mean value of non-desired output is 379.76, with minimum and maximum values of 45.30 and 1272.45 respectively.

**Table 3 Descriptive statistics**

| Variable | Symbol | Mean | Std. | Mix | Max |
|---|---|---|---|---|---|
| Capital | $K$ | 75140.06 | 47867.65 | 12682.15 | 191089.90 |
| Labour | $L$ | 759.67 | 775.34 | 42.87 | 3501.57 |
| Energy consumption | $E$ | 38906.79 | 25306.94 | 5447.70 | 110553.60 |
| Energy output | $EO$ | 16372.59 | 29207.93 | 701.13 | 113160.60 |
| Gross regional product | $GDP$ | 30524.59 | 24060.15 | 2766.74 | 100815.60 |
| Carbon emission | $CO_2$ | 379.76 | 297.16 | 45.30 | 1272.45 |

#### 4.2.2 Overall Efficiency Analysis

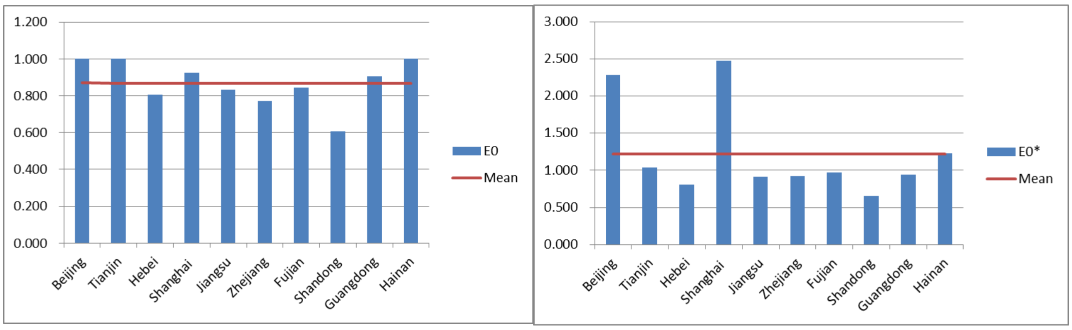
The overall efficiency values based on the conventional and GEEFDEA models are $E_0$ and $E_0^*$ respectively. As shown in Table 4, the ranking of efficiency values under each model is also listed. Under the traditional model, the regions with effective efficiency ($E_0 = 1$) are Beijing, Tianjin, Heilongjiang, Hainan, Sichuan, Yunnan, Shaanxi and Qinghai, which mainly distributed in the eastern and western regions. The main reasons are as follows. On the one hand, for the eastern region, the eastern region has advanced logistics, information, technology and capital flows, and is able to deal with carbon emissions through a advanced technology. On the other hand, for the western region, because the development of industryl is low, the level of carbon emissions is relatively low and the efficiency is high. The three regions with the lowest efficiency values are Inner Mongolia, Jilin and Liaoning, with efficiency values of 0.514, 0.532 and 0.532, respectively. The low efficiency in the regions is mainly due to its low development of economic and technological, as well as the fact that regional development is mainly based on heavy industry. That is, the level of carbon emission is high and the level of technology to deal with pollution is low. As a result, both of energy production efficiency and energy utilization efficiency are low in these regions. For the GEEFDEA model, the average value of efficiency is 1.022 greater than the efficiency value of 0.828 under the traditional model. Specifically, the number of regions with efficiency values $E_0^* > 1$ is greater than the number of regions with effective efficiency $E_0 = 1$ under the traditional model, including Shanghai, Beijing, Qinghai, Shanxi, Hainan, Heilongjiang, Shanxi, Yunnan, Sichuan and Tianjin. Moreover, the carbon emission efficiency values of most regions under the GEEFDEA model are higher than those under the traditional model. Although the efficiency values are not improved in some regions, they also show stability. In general, compared with the traditional model, the carbon emission efficiency in GEEFDEA model with feedback and fixed-sum undesirable outputs based on feedback and fixed-sum undesirable outputs is effectively improved.

**Table 4 Efficiency and ranking of basic model and the GEEFDEA model**

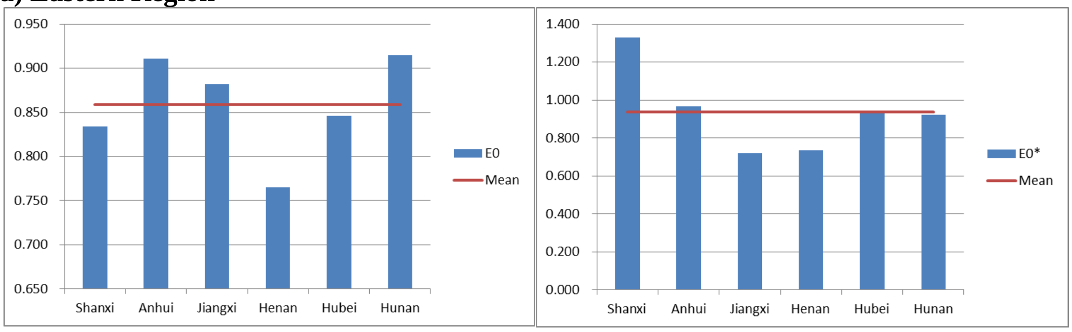
| Region | $E_0$ | Rank | $E_0^*$ | Rank |
|---|---|---|---|---|
| Beijing | 1.000 | 1 | 2.279 | 2 |
| Tianjin | 1.000 | 1 | 1.035 | 10 |
| Hebei | 0.805 | 19 | 0.805 | 20 |
| Shanxi | 0.834 | 16 | 1.329 | 4 |

| | | | | |
|---|---|---|---|---|
| Inner Mongolia | 0.514 | 30 | 0.777 | 22 |
| Liaoning | 0.593 | 28 | 0.555 | 30 |
| Jilin | 0.532 | 29 | 0.609 | 29 |
| Heilongjiang | 1.000 | 1 | 1.203 | 6 |
| Shanghai | 0.924 | 9 | 2.477 | 1 |
| Jiangsu | 0.834 | 16 | 0.910 | 17 |
| Zhejiang | 0.772 | 21 | 0.921 | 15 |
| Anhui | 0.911 | 11 | 0.967 | 12 |
| Fujian | 0.846 | 14 | 0.974 | 11 |
| Jiangxi | 0.882 | 10 | 0.720 | 24 |
| Shandong | 0.606 | 12 | 0.653 | 28 |
| Henan | 0.765 | 26 | 0.737 | 23 |
| Hubei | 0.846 | 14 | 0.941 | 14 |
| Hunan | 0.915 | 10 | 0.921 | 16 |
| Guangdong | 0.905 | 12 | 0.944 | 13 |
| Guangxi | 0.633 | 26 | 0.794 | 21 |
| Hainan | 1.000 | 1 | 1.231 | 5 |
| Chongqing | 0.793 | 20 | 0.834 | 18 |
| Sichuan | 1.000 | 1 | 1.056 | 9 |
| Guizhou | 0.725 | 23 | 0.713 | 25 |
| Yunnan | 1.000 | 1 | 1.076 | 8 |
| Shaanxi | 1.000 | 1 | 1.144 | 7 |
| Gansu | 0.692 | 24 | 0.703 | 26 |
| Qinghai | 1.000 | 1 | 1.848 | 3 |
| Ningxia | 0.834 | 16 | 0.825 | 19 |
| Xinjiang | 0.662 | 25 | 0.683 | 27 |
| Mean value | 0.828 | | 1.022 | |

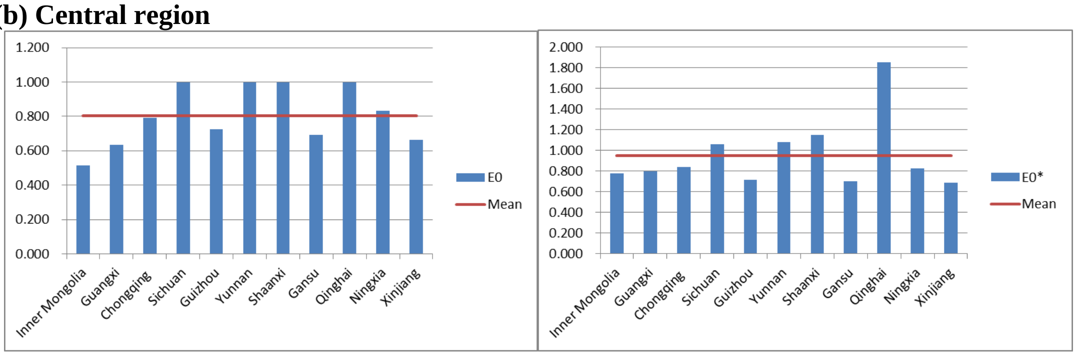
Figure 2 shows the changes in carbon emission efficiency in the eastern, central, western and northeastern regions based on the two models. In Figures 2(a)-2(d), the left and right graphs show the carbon emission efficiency and mean value of each region under the traditional model and the GEEFDEA model, respectively. As can be seen from Figure 2(a), although the carbon emission efficiency and mean value based on the GEEFDEA model is improved in value compared with the traditional model, the number of regions higher than the mean value decreases. From the original five regions of Beijing, Tianjin, Shanghai, Guangdong and Hainan to three regions of Beijing, Shanghai and Hainan. Tianjin and Guangdong move from above average to below average. However, the carbon emission efficiency values of them are still higher than under the traditional model. The values of Beijing and Shanghai break through 2.000, which are 2.279 and 2.477 respectively. From Figure 2(b), it can be found that the carbon emission efficiency calculation based on the traditional model is higher than the mean value (0.859) for Anhui, Jiangxi and Hunan, while the carbon emission efficiency based on the GEEFDEA model is higher than the mean value (0.936) for Shanxi, Anhui and Hubei. Especially, Shanxi province breaks through 1.000 and reaches 1.329, reversing from the fifth rank to the first rank. From Figure 2(c), it can be found that the carbon emission efficiency calculation based on the traditional model is higher than the mean value (0.805) in Sichuan, Yunnan, Shaanxi, Qinghai and Ningxia. The carbon emission efficiency values are all 1.000 except for Ningxia. The carbon emission efficiency based on the GEEFDEA model is higher than the mean value (0.950) in Sichuan, Yunnan, Shaanxi and Qinghai, which are 1.056, 1.076, 1.144 and 1.848, respectively. As can be seen from Figure (d), the mean value of carbon emission efficiency based on the GEEFDEA model (0.789) is improved compared with that under the traditional model (0.708). The carbon emission efficiency of Heilongjiang province has been ranked first in the northeast region. The difference is that the value of carbon emission efficiency based on the GEEFDEA model in Liaoning Province is lower than that under the traditional model. Comparing the four regions, under the traditional model, the average value of carbon emission efficiency in descending order is eastern region (0.869) > central region (0.859) > western region (0.805) > northeastern region (0.708). Based on the GEEFDEA model the average value of carbon emission efficiency is ranked from largest to smallest as eastern region (1.223) > western region (0.950) > central region (0.936) > northeastern region (0.789). Although the ordering of the central and western regions has changed, they have all improved overall in values.



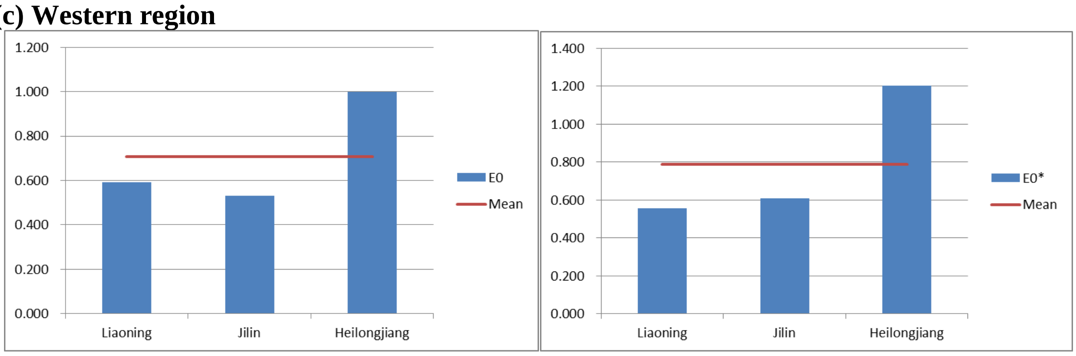

**(a) Eastern Region**




**(b) Central region**




**(c) Western region**




**(d) Northeastern region**

**Figure 2 Carbon emission efficiency of four regions based on the two models**

### 4.2.3 Adjustment of carbon emission efficiency

In order to maximize the energy efficiency of China's 30 provinces and regions in China, this paper analyzes the optimal adjustments to the carbon emission of each region, including the energy production stage and the energy use stage. That is, energy efficiency is improved by increasing or decreasing the carbon emission for each region. When $\Delta CO2 > 0$, it indicates that the region needs to increase the level of carbon emissions to reach the equilibrium efficient frontier. when $\Delta CO2 = 0$, it means that the region remains unchanged at that stage. when $\Delta CO2 < 0$, it indicates that it needs to reduce the level of carbon emissions to realize the efficiency optimization.

As shown in Table 5, in the energy production stage, the sum of carbon emission adjustments is 864.50. On the one hand, there are 18 provincial areas that need to increase carbon emissions. For example, Beijing, Tianjin, Shanghai, Jiangsu, Zhejiang and so on. They are mainly distributed in the eastern developed regions and western backward regions. This is due to the high energy demand in economically developed regions, while the western regions are rich in resources but low in technical efficiency. On the other hand, the number of regions that need to reduce carbon emissions are 12 regions, including Hebei, Shanxi, Shandong, Inner Mongolia and so on. They are mainly heavy industry provinces or resource-based provinces. These areas need to reduce emissions through technological upgrading or energy substitution. In contrast, in the energy utilization stage, the sum of carbon emission adjustments is -864.50. There are seven regions that need to increase their carbon emissions, including Beijing, Tianjin, Jiangsu, Fujian, Hubei, Hainan and Sichuan. These regions are all economically active regions. In general, GDP growth is strongly correlated with energy consumption. Therefore, it is necessary to moderate carbon emission constraints to achieve development. The regions that need to reduce carbon emissions include 14 regions, including Hebei, Shanxi, Inner Mongolia, Chongqing and so on. These regions are mainly industrial provinces with high emissions. In addition, carbon emissions do not need to be adjusted in some regions, including Heilongjiang, Jilin, Liaoning, Yunnan, and Shaanxi. The regions are all at a lower level of development.

**Table 5 Efficiency and ranking of basic model and the GEEFDEA model**

| Region | Energy production stage ( $\Delta CO2$ ) | Energy utilization stage ( $\Delta CO2$ ) |
|---|---|---|
| Beijing | 427.39 | 504.72 |
| Tianjin | 347.01 | 353.61 |
| Hebei | -406.71 | -393.46 |
| Shanxi | -431.52 | -192.77 |
| Inner Mongolia | -431.20 | -113.03 |
| Liaoning | -39.05 | 0.00 |
| Jilin | -273.73 | 0.00 |
| Heilongjiang | -279.24 | 0.00 |
| Shanghai | 318.74 | -123.40 |
| Jiangsu | 290.10 | 504.72 |
| Zhejiang | 91.13 | -284.56 |
| Anhui | -105.04 | -38.90 |
| Fujian | -217.90 | 72.76 |
| Jiangxi | -262.26 | -149.75 |
| Shandong | -1433.18 | -312.40 |
| Henan | 27.02 | -333.66 |
| Hubei | 166.45 | 63.09 |
| Hunan | 199.13 | -81.16 |
| Guangdong | -92.43 | -204.72 |
| Guangxi | 227.66 | -296.27 |
| Hainan | 462.77 | 504.72 |
| Chongqing | 346.65 | -472.23 |
| Sichuan | 194.49 | 168.81 |
| Guizhou | 246.40 | -40.61 |
| Yunnan | 270.72 | 0.00 |
| Shaanxi | 181.28 | 0.00 |
| Gansu | 323.06 | 0.00 |
| Qinghai | 453.57 | 0.00 |
| Ningxia | 275.11 | 0.00 |
| Xinjiang | -11.92 | 0.00 |
| Adjusted value | 864.50 | -864.50 |

## 5.Conclusions and implications

### *5.1 Conclusions*

This paper proposes a two-stage DEA efficiency evaluation model with feedback and fixed-sum undesirable outputs, which reveals the black box process of energy production and energy utilization.

Then, we adopt the GEEFDEA method to construct the general equilibrium effective frontier, which answers the question of how to improve the carbon emission efficiency of 30 provincial regions in China. We also compare and analyze the carbon emission efficiency of 30 provincial-level regions in China with the traditional DEA and GEEFDEA model.

The study found that compared to traditional models, the carbon emission efficiency of the GEEFDEA model has been effectively improved, which is based on feedback and fixed-sum undesirable outputs. For regions, under the traditional model, the average value of carbon emission efficiency is in descending order of eastern region > central region > western region > northeastern region. And the ranking based on the equilibrium efficient frontier surface model is eastern region > western region > central region > northeastern region. Although, the ranking of the central and western regions changed, but in the value of the overall have been improved.

The regions with high carbon emission efficiency values are mainly located in the eastern and western regions of China. For example, Beijing, Tianjin, Sichuan, Yunnan, Qinghai and so on. For the eastern region, it has advanced logistics, information, technology and capital flows, which is able to deal with carbon emissions. This means that the efficiency of both the energy production and energy utilization stages is higher than other regions. For the western region, the level of industrial development is not high, which corresponds to a lower level of carbon emissions. Thus, the value of carbon emission efficiency is high. The regions with low carbon emission efficiency values are Jilin and Liaoning. The main reason for this is the low level of economic and technological development and the predominance of heavy industry. In other words, the level of carbon emissions itself is high and the level of technology to deal with pollution is low.

The substage efficiency decomposition model answers the question of how carbon emission credits should be adjusted at each stage. In the energy production stage, provinces requiring increased emissions are mainly developed eastern regions and underdeveloped western areas, while those needing reductions are predominantly heavy industrial or resource-dependent provinces. In the energy utilization stage, the provinces that need to increase carbon emissions are mainly in the economically active regions, and those that need to reduce carbon emissions are mainly in the provinces with high-emission industries.

### *5.2 Implications*

The policy implications of this study are as follows.

Firstly, resources and energy should be rationally utilized in accordance with local conditions. The eastern region has advanced technologies and talents. It can give priority to promoting the technological research of cleaner and low-carbon for the energy system to play a leading role in other regions, such as the technology of carbon capture and storage. The central, western and northeastern regions are rich in fossil energy, solar energy, wind power and other renewable energy, which can promote the development of clean energy bases and the efficient ustilization of energy. For example, in order to optimize the energy structure and achieve sustainable development of energy resources, the regions increase the development of wind power and photovoltaic power generation bases in areas such as Qinghai, Gansu and Inner Mongolia.

Secondly, strengthening regulation and assessment. Strengthen law enforcement and crack down on illegal emissions and energy waste. At the same time, improve the dual-control system of energy. Reasonably set targets for the intensity and total amount of energy consumption, and strengthen the responsibility of local governments and enterprises to reduce emissions.

Thirdly, government should play a supportive and leading role. On the one hand, to realize cross-regional research and application of technologies, government should set up a shared platform for technology development of energy and low-carbon. This can promote enterprises to realize low-energy consumption and high-technology development. On the other hand, government should increase financial support for the introduction and research of energy technologies, which can provide a positive and healthy environment for the efficient utilization of energy and sustainable development.

Last but not least, it is important to strengthen the public's environmental awareness and behavioral guidance. Popularize knowledge of energy conservation through media, education and other channels. Advocating low-carbon lifestyles, such as saving electricity, green travel and garbage classification.

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