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CONTEXTUAL APPROACH FOR FUNDAMENTAL LAW OF ACTIVE MANAGEMENT

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Abstract

Active Portfolio Management is a simple yet powerful tool for portfolio selection which has became a popular strategy among individual investors. Theoretical background of active management has been established by Fundamental Law of Active Management. The main postulate of this law suggests that in order to achieve additional return alpha, investors should apply a model with higher forecasting power (IC) and should apply it to a maximum number of assets (breadth). Individual investors are attracted by this straightforward and uncomplicated logic and that is the reason why active management is so popular strategy among them. However, the original and later developed variants of the law lead to the same conclusion – for better results the forecasting model must be applied to as many as possible assets. However, this contradicts with the concentration paradox which investors meet in their investment decisions – applying the same forecasting model to maximum amount of assets results in involving unknown assets with diverse characteristics in analysis. With contextual approach we change this perspective. By applying a contextual approach to active management investors can concentrate the forecasting models to a specific group of assets with similar characteristics. We justify the existence of concentration paradox and by using different fundamental contexts to differentiate stocks in different groups, we test fundamental cross-sectional regression models as forecasting models for active investment. With data from Taiwan Stock market we prove that both ex-ante IC and realized return achieved with contextual models outperform the general portfolios which follow the standard postulate of fundamental law.

Keywords

Fundamental Law Of Active Management, Contextual Approach, Active Portfolio Management, Factor Cross-Sectional Models, Investment Breadth

1. Introduction

Stock markets have kept rising in the last few years. This constant growth of the market has made finding alpha returns very difficult. Active investors need to get better and better at finding alpha opportunities. At the same time, with the rise of data analytics nowadays, there is a universe of information related to stock returns. Although at a glance, this seems a positive factor for active investors, in reality, too much information makes it complex and confusing. However, another approach can reduce this complexity and is easily understood by investors. Applying factor models within a specific "context" can deliver excellent results. In practice, this approach suggests splitting the universe of stocks into different "contexts" and finding what factor works for each context rather than trying to find the factor/information that explains the entire universe of stocks. There is sound economic logic behind that approach, which is also supported by the Fundamental Law of Active Management framework. It is counter-intuitive that concentrating on smaller samples of stocks can give better results because investors usually praise diversification. However, such concentration of efforts within contexts helps improve the predictions' accuracy and gives active investors better results overall.

2. Theoretical Review of Active Management

Active investment has always been attractive to investors because of its simple logic – invest when you are convinced in your predictive skills and apply your knowledge as often as you can. However, only after the fundamental work of Grinold and Kahn (G&K) was presented in several papers[1][2][3], the active investment received a theoretical explanation. Active return α is an excess to return provided by those assets in the investor's portfolio that are expected to divert positively from their consensus return. To get an alpha return, investors should increase the weights in their portfolio of these assets, which they score as positive, "good," and decrease the weights of low-scored, "bad" assets. Investors score an asset with their cross-section forecasting skills based on information signals that separate the asset universe cross-sectionally. According to G&K, investors should apply their successful forecasting skills to as many bets as possible, i.e., to broaden the universe of assets multiple times. This approach G&K pretentiously called the Fundamental Law of Active Management (FLAM) and it has become the basic theoretical financial model for active management – (1).

(1) $$IR = \frac{\alpha}{\sigma^*} = IC * \sqrt{Br}$$

where:
IR is the portfolio's information ratio;
IC - the information coefficient (forecasting power) of the investment strategy and $IC = Corr[E(r_i); r_i]$;
Br – the breadth of the portfolio measured as the number of bets per year, i.e., the number of assets and frequency of investment decisions taken over them;
α - active return;
σ^* – active risk.

Equation (1) describes information ratio IR, which is additional active return α achieved for additional active risk taken σ^*. It depends on two factors: a) information coefficient IC – the investor's forecasting skills described by the correlation between forecasted excess return ($E(r_i)$ and realized excess return r_i of an asset, and b) breadth Br, which in G&K variant is number of bets taken by the investor per year, in other words, number of assets to which the forecasting skills are applied multiplied by frequency of the forecasts made over them. According to FLAM, investors, to increase the IR of their investment, should apply their best forecasting skills (those with high IC) to as many assets as possible, which should be done more often (high Br).

Of course, as with every law in science, FLAM answers the question of what happens but does not answer the question of why it happens. The main goal of every investor is to achieve high value added by active management, and according to (1), three elements influence it – IC, Br, and σ^*. In search of more explanatory power and to analyze in depth why such a relation exists, since its appearance the FLAM has been constantly improved in these three elements.

FLAM stimulates investors' quest for strategies and models providing high IC. The first direction of the development of active investment management was to find models for increasing investors' forecasting skills. The logic of active management is to find forecasting signals that can define the future ex-ante performance of the assets and help investors discern outperforming ones. The task of active management is to define which assets will be positively diverted, "good" and which negatively, "bad" from the benchmark. Although FLAM is applicable even for such non-quantitative signals as intuition, the most natural way for investors is to apply factor models. The best way to do so is through scoring and cross-section regression models are perfect tools for that. With cross-section approach, investors can rank all assets they work with according to the available foresting signal j and to run cross-section regression as of (2)[4]

(2) $$\tilde{r}_{i,t} = \gamma_j . f_i + \varepsilon_{i,t}$$

where:
$\tilde{r}_{i,t}$ is the standardized with conditional variance expected active return of stock i at time t;
γ_j– parameter of the cross-sectional regression;
f_i– normalized and standardized ranking of assets i at time t-1 with mean 0 and dispersion 1;
$\varepsilon_{i,t}$– white-noise error.

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[1] Grinold, R. and R. Kahn. "Information Analysis." *The Journal of Portfolio Management*, 18/3, (1992). pp 14-21.
[2] Grinold, R. "The Fundamental Law of Active Management." *The Journal of Portfolio Management*, 15/3, (1989). 30-37.
[3] Grinold, R. & Kahn, R. (2000). *Active Portfolio Management*, 2nd edition. New York: McGraw-Hill.
[4] For more explanation of standardizing the elements of (2) see: Patev, Pl. and K. Petkov. "Comparing Strategy Risk Models on Taiwanese Stock Exchange", *Journal of Wealth Management*, vol. 21, no. 3, (2018), pp. 79-93

In (2), active returns are standardized with estimated conditional variance for stock; f_i also are standardized in such a way to have the cross-section mean 0 and cross-section dispersion 1. With these standardizations, parameter γ_j becomes a cross-section correlation between the forecast and residual returns. If an investor applies the j-factor for its scoring of all stock universe - f_i, the typical cross-section parameter γ_j describes the forecasting power of factor j, and in fact, it presents IC_j of the active strategy using this factor as a signal. In such a way, although the original idea of G&K for IC was to measure the subjective personal forecasting skills of portfolio managers, with the application of factor models, IC naturally evolves into a measurement of the forecasting power of every j-signal - IC_j.

With the vast application of quantitative analysis, portfolio management started testing models in search of characteristics of stocks that better describe them in the cross-sectional realm. Fundamental characteristics of the companies are the most logical indicators that can be used as signals, and these signals can be converted to scoring. This logic is supported both by theory and investment practice. Starting from the general indicators like profitability or operating margins and finishing with liquidity ratios - all of them are supposed to influence investment decisions and market performance of the stocks. Fundamental factor models have become the most intensively applied in active management instruments for scoring because the fundamental data is available, standardized, and reliable. As a result of the application of fundamental factors in the cross-sectional dimension of active management, investors are stimulated to search for the fundamental factors that provide higher IC_j and apply these factors to a broader universe of assets.

The second direction of research development of FLAM was to improve the understanding of active risk σ^*. The active risk is incurred when managers deviate from the benchmark. To achieve additional to their benchmark return, investors should deviate from it and endure additional risk – tracking errors, which the original FLAM presented as active risk. Initially, G&K assumed IC to be constant, meaning that the forecasting power of factor models is assumed to be unchanged during the time. Later, the original FLAM was extended to allow a variance of the forecasting skill through time. First, Qian and Hua (2004)[5] pointed out the importance of the volatility of IC - σ_{IC} as an additional source of active risk. They call this strategy risk and suggest how it should be considered in evaluating σ^* . While Qian and Hua presented the additive character of σ_{IC}, Ye (2008)[6] describes the strategy risk as multiplicative for the active risk[7]. However, the full model of active risk involving strategic risk has been developed by Ding and Martin (D&M)[8][9] - (3)

$$IR = \frac{IC}{\sqrt{\frac{1-IC^2-\sigma_{IC}^2}{N}+\sigma_{IC}^2}} \tag{3}$$

Active risk consists of two distinct parts – tracking error and strategy risk. Tracking error is the natural volatility of excess returns around their benchmark, and it is usually managed by good diversification. On the other hand, strategy risk is the risk that the selected factor model does not properly forecast the excess returns. However, both versions assume a non-existent relationship between IC and the number of assets.

The third direction of active investing is the development of Br. FLAM from (1) motivates investors to search for strategies with high IC and stimulates them to apply these models to the more extensive universe of assets as, according to (1), they have to increase the breadth Br. In the original model of G&K, Br was involved as a number of bets that investors take. However, when FLAM is applied with factor models, the frequency of signals appearing is limited (releasing financial statements, for example). Investors often are not in a situation to increase frequency in order to increase investment bets. The only way to increase the breadth is by increasing the number of assets involved in the scoring process. Therefore, with factor models, the breadth becomes the number of assets N, and FLAM motivates investors to increase this number in order to achieve higher IR. Later research developments proved that breadth should not be associated only with the number of assets but with other elements as the correlation in forecasts should be involved – see Buckle (2004)[10]. Although the number of assets is still predominated as a factor to increase the breadth, investors should consider correlation in factors. For example, our

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[5] Qian, E. and R. Hua. “Active Risk and Information Ratio”, *The Journal of Investment Management,* Vol 2, No. 3, (2004), pp. 20-34.

[6] Ye, J. “How Variation in Signal Quality Affects Performance.” *Financial Analysts Journal,* Vol. 64, No. 4, (2008), pp. 48-61.

[7] See Patev, Pl. and K. Petkov. “Comparing Strategy Risk Models on Taiwanese Stock Exchange”, *Journal of Wealth Management*, vol. 21, no. 3, (2018), pp. 79-93

[8] Ding, Z., D. Martin and C. Yang. “Portfolio Turnover when IC is Time Varying”, *Journal of Asset Management,* vol. 21, (2020), pp. 609–622.

[9] Ding, Z. and R. D. Martin. “The fundamental law of active management: Redux.” *Journal of Empirical Finance*, 43/3, (2017), 91-114

[10] Buckle, D. “How to calculate breadth: An evolution of the fundamental law of active portfolio management.” *Journal of Asset Management*, 4/6, (2004), pp. 393–405.

previous study[11] proven that optimal IR can be achieved with a relatively small number of traded stocks (5% for the Taiwanese stock market). However, investors still must forecast alphas for the whole universe of stocks before involving correlation of forecasts and start finding the optimum number of assets in their portfolio.

As a result, the three directions of FLAM developments – IC, σ^*, and Br- modify the central postulate of active management: When it is applied to fundamental cross-section factor models, FLAM requires investors to search for factors that (a) provide higher IC, (b) experience less volatility σ_{IC}, and (c) apply them to all universe of assets (or as many as possible asset, i.e. maximize N).

However, such an approach excludes the specifics of characteristic dimensions of which the models work. Since the first Fama and French study, thousands of papers have proven that dimensions such as size, value/growth, and momentum definitely lead to the clustering of the market and make factors influence non-general over the whole universe of stocks. Therefore, another approach – a contextual approach should be applied in forecasting. Sorensen, Hua and Qian (SHQ)[12] , contributed to development of active management by their contextual approach. The simple idea of the contextual approach suggests that alpha factors work better when applied to specific types of stocks (contexts) since, within those groups, stocks have distinctly different characteristics, and it is logical that the same factor would not be able to explain excess returns in very different stocks.

It seems that the contextual approach contradicts the requirements of FLAM in cross-section forecasting models. Working with the entire universe of stocks theoretically will increase breadth. However, the contextual approach suggests splitting the universe of stocks into smaller groups (context), which lowers breadth. Furthermore, the contextual approach relies on increasing IC (forecasting ability) within the smaller groups. However, FLAM assumes no relationship between IC and Br, it does allow for time-series variance of IC, but not across different number of stocks. FLAM infers that the IC of the model will be the same when applied on 100 as on 500 stocks. It is very restrictive and does not hold in real-world, this is where contextual approach adds value.

3. Concentration paradox

In order to incorporate the contextual approach to active investment management, we first direct our attention to the concentration paradox that investors meet when they apply FLAM. The central postulate of FLAM is that if the investors aim to increase α, they should apply their IC to higher breadth to increase N. However, this is valid only with the assumption that IC is constant for all assets. The paradox is that practically, investors know that by increasing the number of assets in their portfolio, the forecasting power of their model is being diluted. Investors have the opposite of FLAM's suggestion behavior – if they find that some of the forecasting models work correctly, they try to apply them to stocks they know better. There is an expectation that the predictive power of the models will not work equally among all stocks. FLAM focuses on the diversification effect of increasing N because it leads to a decrease in active risk. In its variations, the assumption of constant IC has been kept. However, as we can see, increasing N may have a negative effect on IC. We can show that in both the primary (1) and advanced (3) concepts of FLAM, the IC will be increasing with the concentration of the strategy over a smaller number of assets. IC for single-factor cross-section models in active portfolio management is estimated as the correlation between standardized factor loading and standardized return. The formula of correlation can be broken down to (4):

(4) $$IC = \frac{cov(r_i, f_i)}{\sigma_{r_i} * \sigma_{f_i}}$$

Further developing (4) into (5):

(5) $$IC = \frac{(\sum(r_i - \bar{r}_i) * (f_i - \bar{f}))/N}{\sigma_{r_i} * \sigma_{f_i}}$$

Since the factor loading in (5) is normalized and standardized, $\bar{f}$ is equal to 0, and the $\sigma_{f_i}$is also 1. Similarly, the return is standardized to have $\sigma_{r_i}$of 1. Additionally, in the cross-sectional model, the average excess return $\bar{r}_i$ is close to 0. Thus, equation (5) can be simplified to (6):

(6) $$IC = \frac{\sum r_i * f_i}{N-1}$$

Plugging IC into the original version of FLAM (1) will lead to (7):

[11] Patev, Pl and K. Petkov. "Risk Adjusted Breadth in Active Portfolio Management.". *International Journal of Business & Management Studies*, vol. 2, no. 7, (2021), pp. 72-82

[12] Sorensen, E., R. Hua, and E. Qian, "Contextual Fundamentals. Models and Active Management." *The Journal of Portfolio Management*, Vol 32, No. 1, (2005), pp. 23-36.

$$IR = IC * \sqrt{N}$$

(7) $$IR = \frac{\sum r_i * f_i}{N-1} * \sqrt{N}$$

Rearranging (7) to isolate the impact of N, we come up with (8):

(8) $$IR = \sum r_i * f_i * \left(\frac{\sqrt{N}}{N-1}\right)$$

Equation (8) shows the negative impact of increasing N on the overall performance - IR will decrease with the ratio $\frac{\sqrt{N}}{N-1}$. This decrease can be visualized in Figure 1.

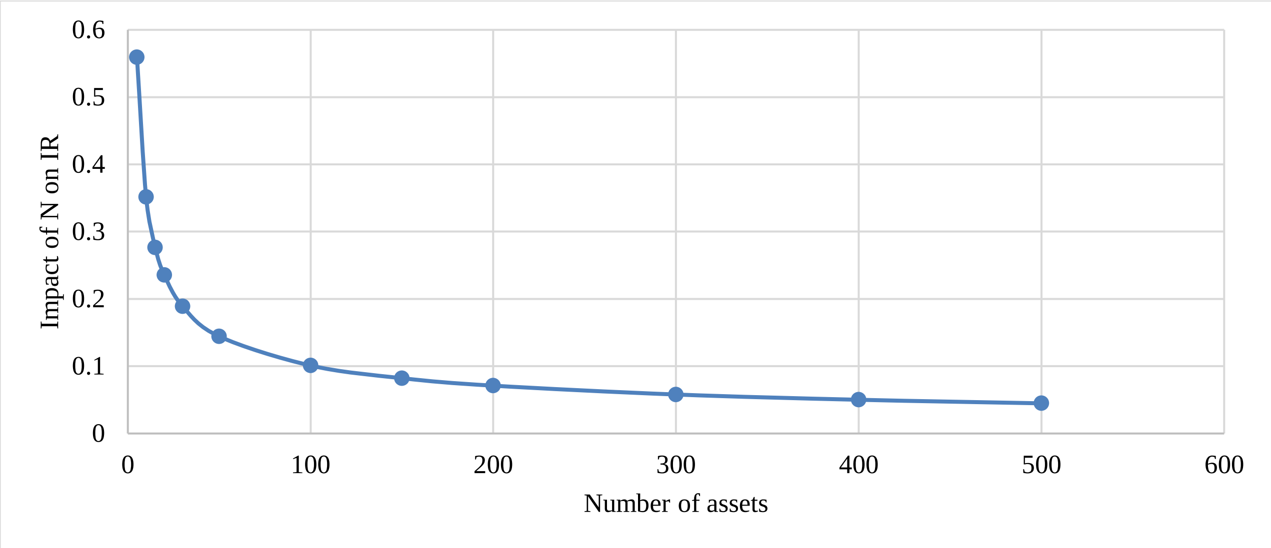

Figure 1: Concentration effect in standard FLAM

The concentration on a smaller number of assets will increase the accuracy of the factor strategy and drive the IR up. Figure 1 perfectly presents the concentration paradox that individual investors meet – by involving more and more assets in their factor model with the hope of increasing alpha; they actually penalize the IR of their forecasting model with $\frac{\sqrt{N}}{N-1}$.

Because of the concentration paradox in (8), investors intuitively concentrate the application of their forecasting model to fewer assets. However, the concentration of portfolio strategy on a small number of assets increases the risk as the diversification benefit is lost. The simplified version of FLAM, as presented in (1), does not represent this effect in an analytical view. Because of that, it is necessary to elaborate on the concentration effect on D&M's form of FLAM, where the diversification benefit is involved. The D&M's version of FLAM (3) gives us another inside look at the concentration effect, this time from the perspective of strategy risk. The only deficiency of equation (3) is the assumption that IC will remain constant when more assets are included in the portfolio. Inputting the new dependency of IC through the time σ_{IC} into equation (3), we get (9):

(9) $$IR = \frac{\frac{\sum r_i * f_i}{N-1}}{\sqrt{\frac{1 - \frac{\sum r_i * f_i}{N-1}^2 - \sigma_{IC}^2}{N} + \sigma_{IC}^2}}$$

The equation (9) can be broken into two parts that provide greater analytical understanding:

i. Forecasting ability IC in the form of $\frac{\sum r_i * f_i}{N-1}$ – as already proven, the forecasting ability is inversely related to the number of assets; therefore, increasing N will decrease the nominator.

ii. Active risk $\sqrt{\frac{1 - \frac{\sum r_i * f_i}{N-1}^2 - \sigma_{IC}^2}{N} + \sigma_{IC}^2}$ – inputting the new notion of N does not impact the general logic of active risk as σ_{IC}^2 is a very small number. Thus, increasing N will have the expected diversification effect on active risk by reducing it.

Overall (9) shows that increasing N will decrease both parts of IR - the forecasting ability IC (nominator in i.) and the active risk (denominator in ii.). The chart plots the effect of N on both components of the IR in Figure 2.

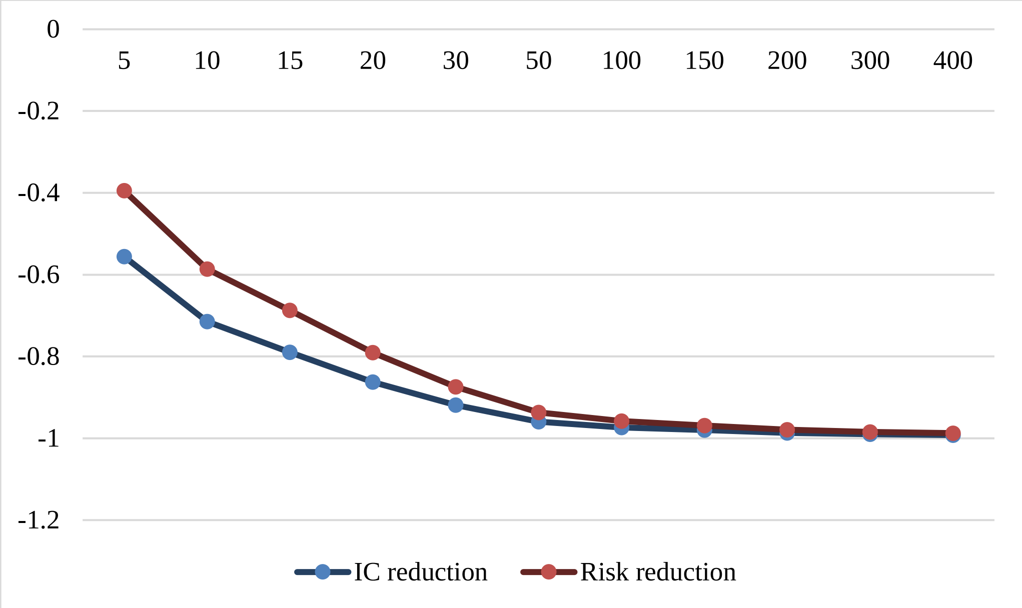

Figure 2: IC & Risk reduction with uncresing of N

Figure 2 explains that while reducing risk has a positive effect, the negative impact on IC is much more significant. In essence, this means that if investors are including more and more assets in their active portfolios, they are losing their forecasting ability as it is harder to find models that predict accurately such a large number of assets, while on the other hand, they are decreasing their active risk. However, with a sufficiently large N, both effects negate each other. Such a large N is not applicable for small investors because having more than 300 assets is possible for large institutional investors. The overall effect on IR with the added impact on IC is shown in Figure 3.

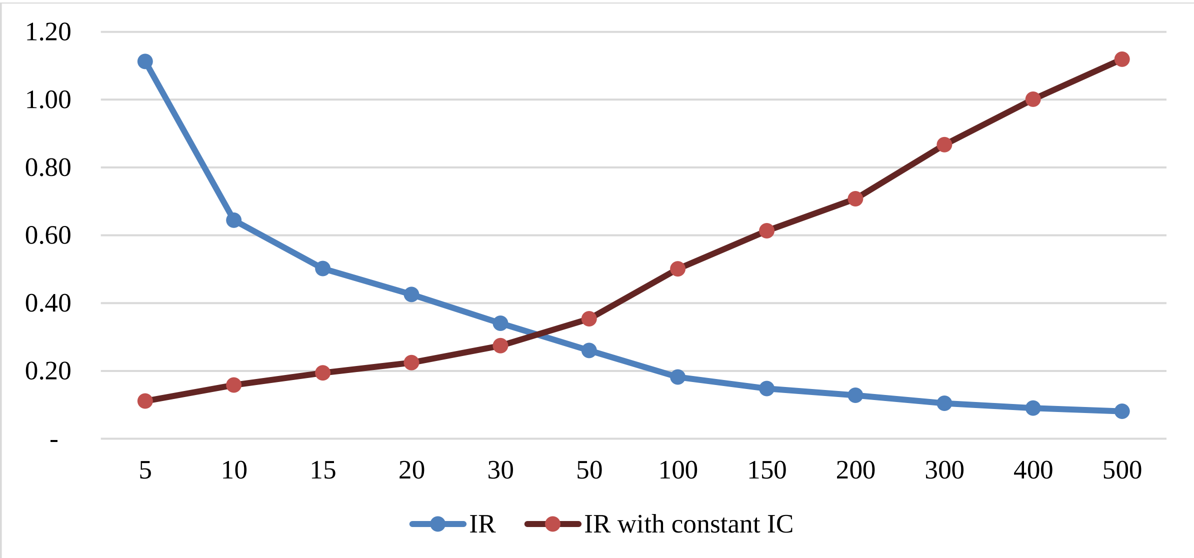

Figure 3: IR with constant and changeble IC

Both (1) and (3) variants of FLAM are based on assumptions of constant IC among the assets involved in the asset universe. This implies that the model's forecasting ability will be the same as that of all assets involved in the investor's universe. That assumption gives the false impression that IR will continue to grow with increased assets. Figure 3 explains the difference in results for IR when the assumption is valid and when it is relaxed. Relaxing that assumption shows that, in fact, IR will be decreasing toward 0. Such a result is entirely in accordance with the intuition that individual investors follow – it is natural to expect that no investor will have the skill to forecast 500 stocks as well as 15.

After relaxing the assumption of constant IC, we found that the postulate of active management should be changed dramatically. Instead of as a requirement for maximum increasing N in the investor's universe in which IC is applied, FLAM can now be interpreted as a requirement for concentration to that part of the universe where IC is still high enough.

4. Contextual Approach

The contextual fundamental model applies factor strategy in a particular market section. As a result, the strategy will be applied to fewer stocks, which lowers the breadth. However, the idea behind the contextual model is that the factor will perform much better with a smaller sample of companies. This is not only due to the concentration paradox, but it is also based on economic logic.

Factor models rely on correctly predicting the distribution of returns based on their own distribution. For example, investors expect companies with high margins to have higher returns. For this to happen, the market must price high-margin companies better than low-margin companies. Based on this logic, investors who aim to get higher returns will observe which companies have high margins and will buy their stocks to take advantage of the process. However, that approach has an issue – not all companies are made equal. Since the time of Fama&French's three-factor model[13], we know that the market does not evaluate the whole universe of stocks similarly. Simply put, there are different types of companies.

- Industry – companies from different sectors have inherently different margins. For example, Tech companies are very profitable and have high margins; on the other hand, consumer staples companies rely more on volumes and have low margins. Therefore, comparing Tech company and Consumer Staple based on margin makes little sense. Furthermore, for the consumer staple company, the margin factor may not be necessary at all because the market knows this factor is typically low, while a factor based on revenue growth will be much more critical.
- Size – the same argument can be made about companies with different sizes. For the pricing of smaller companies, it is much more relevant to have higher margins that will attract investors. Similarly, for larger businesses, it is essential characteristics like return on assets in order to see if the scaling and expansions go well.
- Stage of the life-cycle – companies that are in their "growth" phase of development will have inherently higher margins, which makes the factor relatively insignificant for them, but factors like return on equity will be significant because profitability and high growth will attract investors. On the other hand, overpriced companies will need to deliver high margins to show investors they still have an effective business.

Understandably, the same factor will not work equally well for different types of companies. It will be essential for some companies, and this factor must be included in the pricing model, while it will be insignificant for others. Applying the factor over the entire population of stocks will, therefore, be a mistake for the investor. A better approach will be to split or "slice" the investment universe into groups and apply the factor to the appropriate group. Such calibration can improve the performance of the selected factor in stock picking and deliver rather significantly higher returns for investors

SHQ have already shown evidence that the contextual approach works in practice. The goal of this paper is to answer the question of why the contextual approach works while contradicting several assumptions of FLAM. In order to achieve this, the research looks at the economic logic behind contextual investment as well as the concentration benefits that turn out to have a significant impact on IC. We slice our universe across only in two contexts – VALUE and SIZE. We do not test MOMENTUM context as it requires higher frequency data and fundamental factors are rarely used for active portfolio investments.

5. Data description

The testing is performed on the stocks from Taiwan Stock Exchange. The largest 600 stocks by market cap are selected to be part of our stock universe to ensure that all have data for the last ten years from Jan 2013 till the end of 2023—the source of information about stock prices from TEJ+ databank. We apply the eight most popular alpha factors (explained in Appendix 1) used for active portfolio selection by individual investors. The database is again TEJ+ databank - quarterly financial statements.

To form the contextual groups, the set of stocks is sliced across the VALUE and SIZE contexts. The VALUE context slices the market in two groups – "Undervalued group" and "Overvalued group" based on the relation of each stock's P/E against the corresponding sector's median – those with higher than the average P/E of the sector are involved in "Overvalued group" and those with lower than sector's average – in group "Undervalued group." For the SIZE context, the same set of stocks is sliced based on the market capitalization; in this case, there are three groups – "Large-cap group," "Medium-cap group," and "Small-cap group." All stocks are distributed among the three groups and thus have equal number of assets.

[13] Fama, E. and French, K. (1992), "The Cross-Section of Expected Stock Returns". *Journal of Finance*, Vol. 47, No 2, pp. 427-465.

As first stage part of our research, we find out how the contextual approach can improve each factor's forecasting power (IC). To do so, we apply eight cross-section single-index regressions between the quarterly return of stocks and the eight factor indicators for the same quarter. Factors load and stock returns are standardized, and the IC of each factor is established according to the equation (6). Once we run the regression for all universe of 600 stocks and after that – for the groups in each context. That allows us to compare the ex-ante Raw IC for the entire stock population with the ex-ante IC of each group in that context. The difference between the Raw IC and the best IC among the contextual groups gives us the benefits of the contextual approach.

In second stage of our research, we examine the realized returns from the contextual approach in active management, we track the performance of portfolios selected by the eighth alpha factor. We observe the returns of portfolios constructed within the context and compare them with the performance of active portfolios constructed from our whole stock universe. To do so, in each quarter, we have to rank the stock according to the selected factor, expecting that stocks with higher scores will deliver higher abnormal returns. Again, the scores must be standardized for a mean of 0 and a standard deviation of 1.

As a first step, we have to construct our General portfolio build on a factor models which consider all 600 stocks. We separated the stocks into three quantiles. The top quantile, 1/3 of the stocks in the population, are those we expect to outperform during the next quarter. We construct equally weighted portfolios with these stocks from the top quantile. Each month, the process is repeated, and the portfolio is rebalanced. When we build our General portfolios, we divide the stock universe of 600 stocks into three quantiles based on the score extracted from the alpha factor – top, middle, and bottom. Therefore, our General portfolio consists of the top 200 stocks according to the selected factor.

As the second step for investigating the realized return we have to construct contextual portfolios. When we construct portfolios in the VALUE context, we have already separated the entire set of stocks into two contextual groups – “Undervalued group” and “Overvalued group," each consisting of 300 stocks. We select the top quantile of each group (i.e., 100 stocks that present the top 1/3 of each group) and construct an equally weighted portfolio, which we compare with the General portfolio. These are two examined VALUE context portfolios – Undervalued and Overvalued portfolios. For the SIZE context, the stocks have been distributed among the three contextual groups – “Large-cap group," "Medium-cap group," and "Small-cap group” – 200 stocks in each. We distribute each group into three quantiles and select the top quantile of each according to the factor score. This results in 67 top stocks of each group, and we build an equal-weighted portfolio with them. These are our Large-cap Portfolio, Medium-cap Portfolio, and Small-cap Portfolio, which we compare with the General portfolio.

6. IC And IR Improvement with Contextual Approach

Both the equational interpretation of active management and the economic logic lead us to the presumption that individual investors should not apply alpha factors across all stocks but instead should focus each factor on a given context or sub-group of stocks. Concentrating the portfolio on only part of the asset's universe could increase the forecasting ability of the factor models. However, reducing the number of assets taken into consideration may potentially increase the active risk due to a lack of diversification. In this section, those effects will be investigated with real-world data from Taiwan stock market to examine the presumption that active factors work better in a given context rather than a full set of stocks available.

In the first stage of our research, we present the difference in IC and IR between a general group of stocks involving all 600 stocks in our set and the contextual groups of stocks. We test 8 factors by applying a single-factor model for both the general group and the contextual groups. For every model we test, ex-ante IC measures the forecasting power of each factor – can this factor be used as a predictor for expected returns? Investors are looking for factors that can improve their forecasting ability. The main idea behind the contextual approach lies in the supposition that the factor's information coefficients will increase when applied not on the entire sample of stocks but within each context. We expect that by applying single-factor models to the groups in each context, the model's forecasting power (IC) will be different among the groups. For the purposes of this study, we exclude the analysis of the reasons why one group produces higher IC at some moment as it is a matter of strategy risk. Here, we only test if the context approach can improve IC. Because of that, we compare the highest IC provided by the groups in each context with the general IC provided for the entire universe without a contextual approach.

We separate our stock universe into contextual groups to investigate IC and IR. VALUE context slices the sample into an "Overvalued group" and an "Undervalued group”, and it is expected that for one of them factor models will produce a higher IC than the other for each studied factor. For every quarter, we calculate the difference between the IC of the entire set of stocks (“Raw IC”) and that group’s IC, which is the maximum at that moment. The average difference for the whole period for all eight factors is given in the left side of Figure 4a. As we can see, for every factor we analyze, there is an improvement in contextual IC. Although this improvement may seem not too big for some factors – 0.005 for *Cash Ratio* or 0.009 for *Current Ratio* and *Net Margin*, for other factors, however, the improvement is significant - like *Net Margin* with 0.03 improvement, *ROE* – 0.028 and *ROA* – 0.035 increasing in IC.

Similarly, we compare the contextual IC to the SIZE context. The difference between Raw IC for the entire set of stocks and the best IC provided by some of the three groups in this context is presented on the right side of Fig.4a. For factors such as *Operating Margin*, *Net Margin*, and *ROE*, the contextual IC is improved by close to or higher than 0.030.

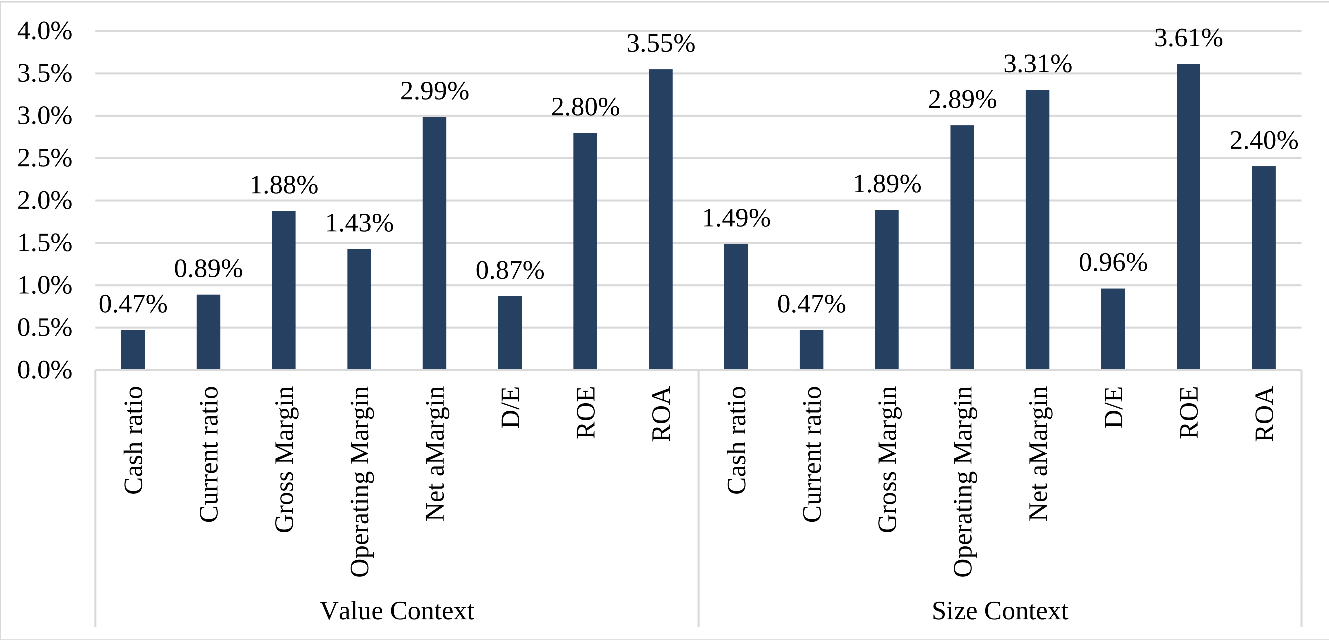

Figure 4a: Change in IC with contexual approach

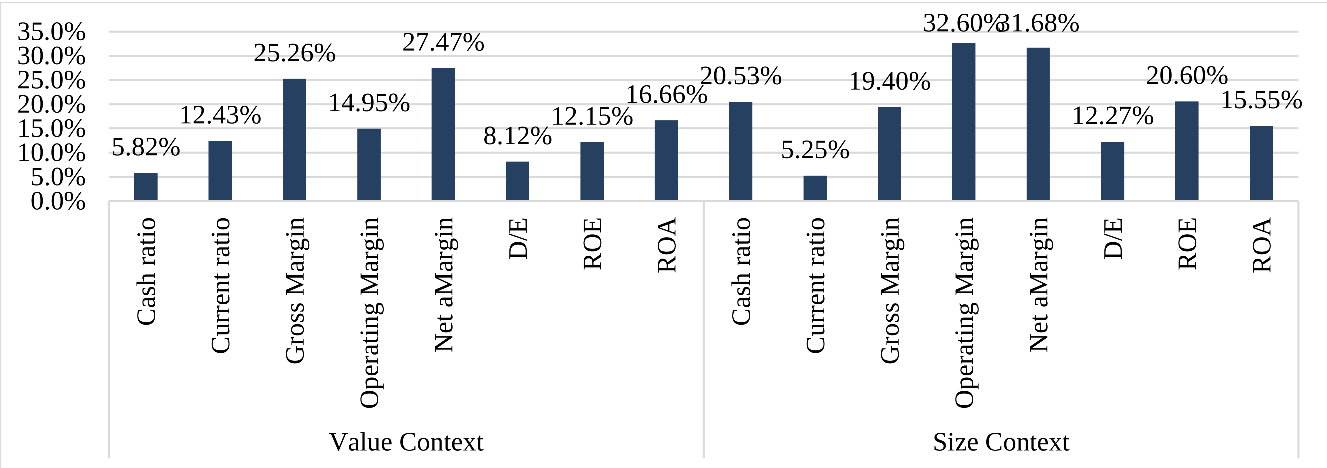

Figure 4b. Change in Ex-ante IR with contexual approach

Of course, the risk in the contextual approach also increases. However, when we combine the context's influence over IC and the active risk – Fig 4b, for most of the factors, we observe a significant increase of IR in the contextual aspect. For both the VALUE context and SIZE context, we prove that increasing is positive for all factors but especially for margins (*Gross* and *Net Margin* for VALUE and *Operating* and *Net Margin* for SIZE context improve IR with more than 0.02). In Appendix 2, we present detailed information about all factors we test as forecasting alpha factors. For each factor it is estimated the Ex-Ante IC when applied in each context and as well as the full sample. Almost in all factors there is significant increase in IC when the factor is applied in the appropriate dimension. This is key evidence that the relationship between IC and number of assets is not constant and IC can be increased when factors are applied in an appropriate context.

7. Return with Contextual Approach

For individual investors, improvements in IC need to be translated into realized returns. To do so, we calculate the ex-post return achieved with application of contextual approach. In order for realized returns to be observed equally, weighted portfolios are created based on factor scores. We construct general and contextual portfolios.

General portfolios are selected with factor models on the whole universe of 600 stocks, while contextual portfolios are selected only on the stocks within the context. This technique closely resembles how investors will deploy contextual portfolio ideas in real-world investing.

Here we use *the Operating Margin* factor as an example. First, we apply this factor to the entire sample to create an equally weighted portfolio of top 100 stocks. This is our General portfolio. Then, using the VALUE context, two more portfolios are created with this factor – the Overvalued portfolio and the Undervalued portfolio. In essence, the Overvalued portfolio contains the high P/E stocks with the highest operating margin, while the Undervalued portfolio contains the low P/E stocks also with the highest operating margin. Figure 5 illustrates the benefits of using factors within the context approach.

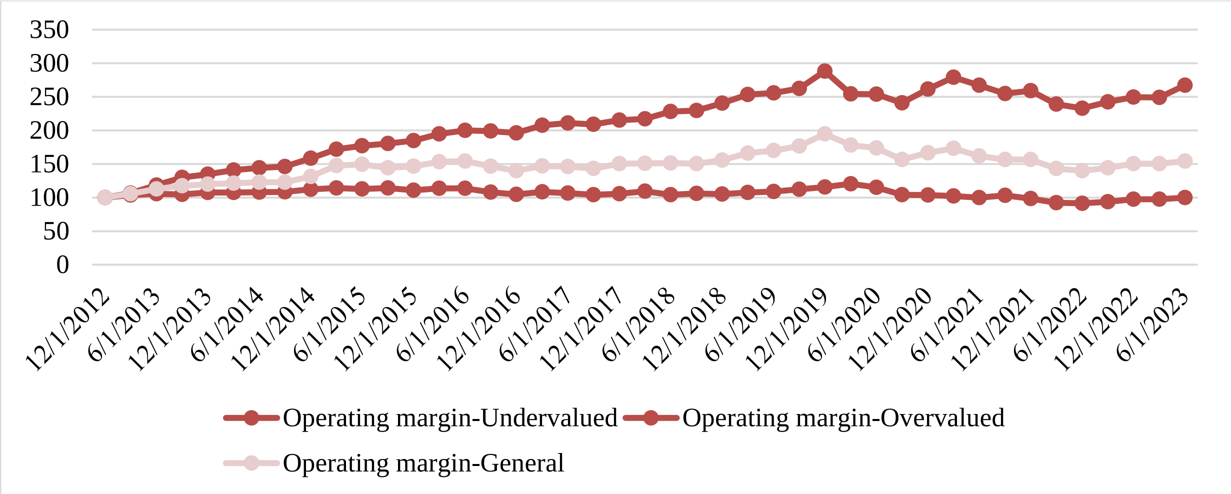

Figure 5. Diference in realized return with contexual approach

Using the factors within a context should lead one of the contextual groups to outperform while the other one will perform poorly. The chart shows that the Undervalued portfolio performs constantly better than both the Overvalued portfolio and the General portfolio (constructed on all stocks). This is evidence that the undervalued companies with high operating margins have significant positive alpha. From an asset pricing perspective, this means that the market prices are higher than the *Operating Margin* factor forecasts only in Undervalued stocks, while for overvalued stocks, this factor is not that important, thus the poor results of the Overvalued portfolio.

Performing this procedure on all the factors allows us to gather further evidence of the benefits of using alpha factors within the contexts. Figure 6 shows how portfolio returns will improve if factors are applied in the most appropriate group of the context. Following the methodology for IC, the best context for each factor is taken to compare against the portfolio created based on the total sample. The results show a quite significant improvement in each factor's performance. The full results are given in Appendix 3.

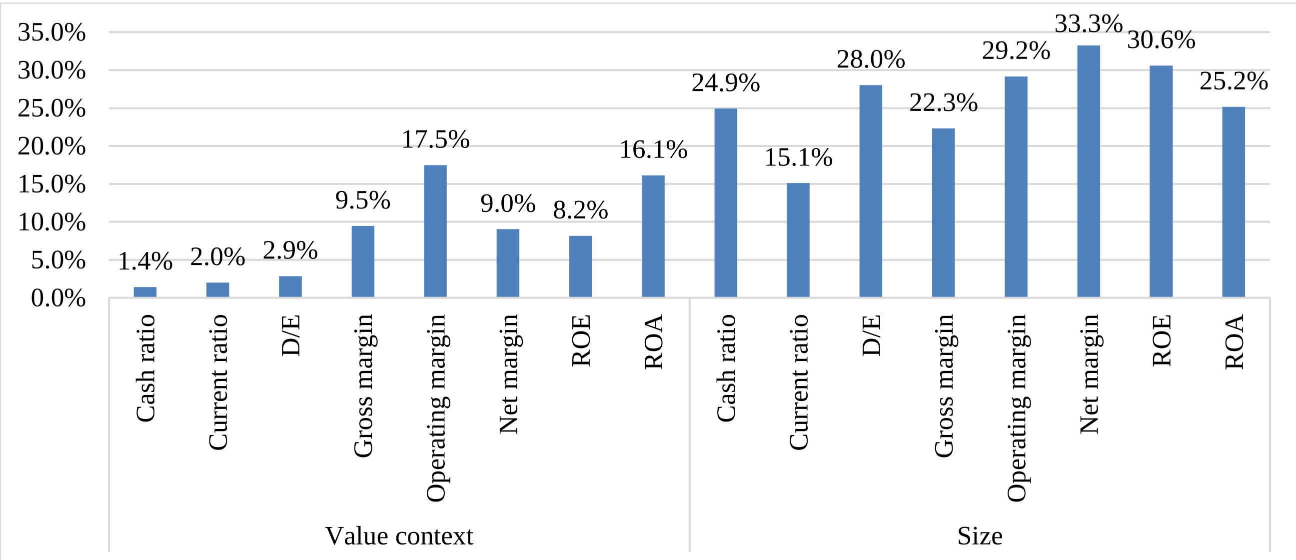

Figure 6a. Improving Portfolio Return Performance with Contexual Approach

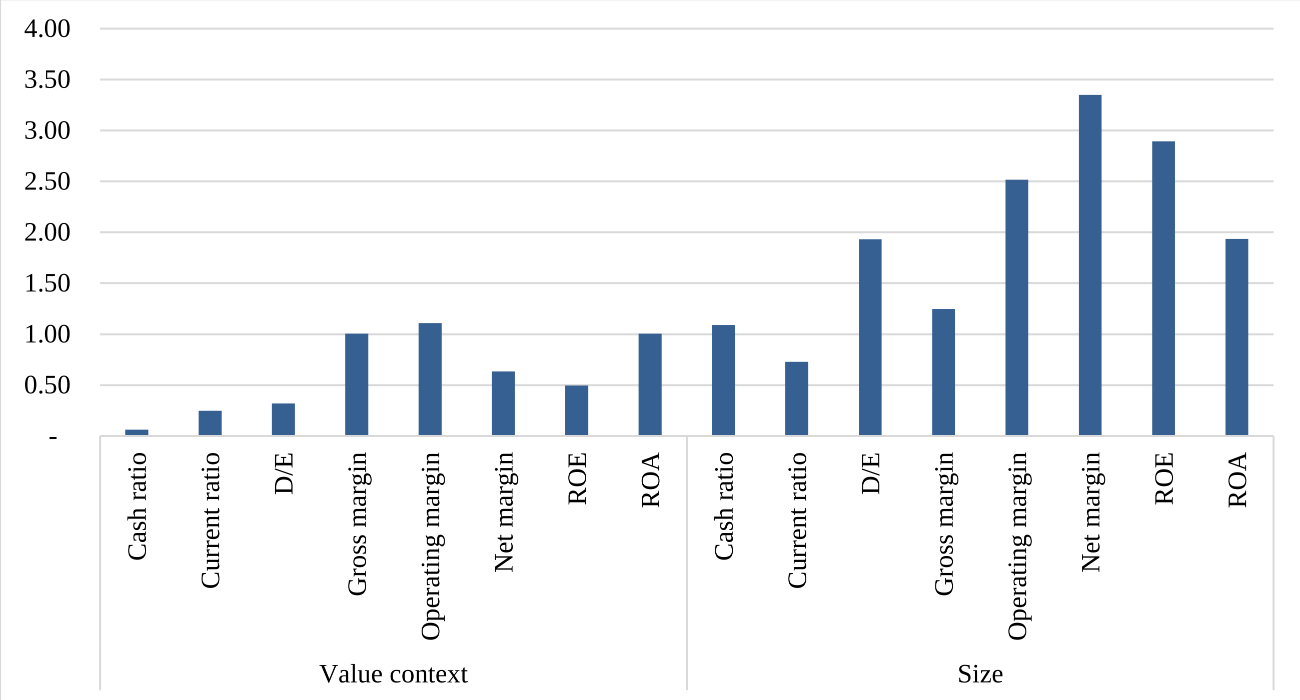

Figure 6b. Improving Portfolio Ex-post IR Performance with Contexual Approach

The results presented in Figure 6 clearly indicate the advantages of deploying alpha factors within a given context rather than analyzing the full universe of stocks. There is increasing in both observed volatility and variation in forecasting ability, but the increase in the ability itself (IC) is far more positive returns outweigh those negativities.

From our results in Figure 6, we can point out several significant findings:

1. For all factors, we observe some improvement in both realized returns. This is evidence that applying the contextual approach can improve portfolio performance. This result is especially important for individual investors, who usually are restricted in the opportunity to increase the number of stocks in their portfolios. In some cases, the improvement can be significant. For example, for factors such as *Net margin* and *ROE* in the SIZE context, improvement in realized return could be above 30%.
2. Contextual application leads not only to high returns but also increases realized IR. Realized IR can be interpreted as a risk-reward ratio equivalent to the Sharpe ratio in active portfolio management. Figure 6b confirms that active return achieved per unit active risk taken is better when factor models are applied to conceptual groups instead of the general universe of stocks, not only from the perspective of increasing returns but also from the perspective of optimizing risk-return characteristics of active portfolios.
3. It is obvious that the contextual approach works better for SIZE context. Slicing the stock universe in this context can give investors more insight into how to forecast better and realize their forecast in return. Therefore, cross-section fundamental analysis based on financial statements is a powerful forecasting tool, but when it is applied in a SIZE context, its efficiency can be increased.
4. As expected, for both contexts, the factors that not only can increase the forecasting power (see Figure 4) but also convert this power nto realized return and realized IR are margins. This result is logical as margins are a good indicator of the companies' business efficiency. Of course, here we observe some important details. In the VALUE context, better improvement can be achieved with *the Operating Margin* and *Gross Margin* for the "Undervalue" group; for SIZE, it is *the Net Margin* and *Operating Margin* for the "Medium"-sized group.

Our results significantly revise the central postulate of FLAM. As all variants of FLAM suggest that the breadth, the number of assets involved in the forecasting model, should be maximized in order to achieve better, the results of our study suggest that these models should be applied in the contextual realm. Not maximizing N is the best way to improve cross-section alpha factor models' forecasting power (IC). We prove that applying the same factor models to smaller groups of stocks in contexts can increase IC and realized active return and IR.

8. Conclusion

The benefits of using alpha factors in a given context (smaller group of stocks) over a full sample of assets are clear. In this case, having "more data" is not as helpful as initially thought. Trying to analyze the entire sample of stocks leads to a significant decrease in forecasting ability, which cannot be compensated by the diversification benefits when including more stocks in the analysis. One of the main postulates of traditional active portfolio management states that increasing breadth (number of assets) in the model leads to increased value added. However, this has the rather significant assumption that an investor's forecasting ability will be the same when forecasting 600 assets as it is when forecasting 67 assets, which is clearly untrue. The effects of concentrations need to be examined more deeply. In essence, contextual investing tells investors to focus on understanding the nature of the stocks being forecasted rather than getting more and more data on all of the stocks while assuming all have similar characteristics. Factors significantly improve their performance only when deployed over the proper context, and this is necessary knowledge that investors must obtain; otherwise, the performance will be marginalized.

Appendix 1

We work with the most popular among individual investors fundamental factors for selecting stocks and investigate their forecasting power (IC) with single-factor models. The way we construct the factors is presented in Table A1.

Factor	Type	Formula	Prediction
Cash ratio	Liquidity	$\frac{Total\ cash}{Current\ liabilities}$	High liquidity ratios predict a stable company; therefore, investors can expect higher returns.
Current ratio	Liquidity	$\frac{Current\ Assets}{Current\ liabilities}$	
Gross Margin	Efficiency	$\frac{Gross\ profit}{Revenue}$	Margins are one of the most important predictors of stock returns. Higher margins mean the company can turn more of its generated revenue into profits.
Operating Margin	Efficiency	$\frac{EBIT}{Revenue}$	
Net Margin	Efficiency	$\frac{Net\ Income}{Revenue}$	
D/E	Capital Structure	$\frac{Total\ Debt}{Equity}$	Capital structure is not directly related to stock performance, but it is an important metric to ensure the company is stable.
ROE	Profitability	$\frac{Net\ Income}{Equity}$	Profitability directly impacts the performance of stock; more profitable companies generate more income for their shareholders, and therefore, it is expected to have higher returns.
ROA	Profitability	$\frac{EBIT}{Assets}$	

Table A1. Fundamental factors for cross-section regression models

Appendix 2

We evaluate ex-ante IC and IR of each contextual group of shares with the IC and IR achieved with all 600 stocks we work. The results are shown in Table A2. All the panels of Table A2 show that factor ICs in contextual group are far greater than the full sample IC. Even factors with traditionally low IC see significant improvement. At the same time, there is a similar increase in σ_{IC} across all factors in the contextual environment. Thus, in reality, the increase in forecasting ability comes with a price of more uncertainty within that ability. This is a consequence of the forecasted lower number of assets. The Information Ratio formula gives great insight into the efficiency of the decrease in assets forecasted. Panel B. shows that there is a positive increase in ex-ante IR[14] across all factors, meaning that the benefit of increasing forecasting ability is greater than the associated risk of concentration.

[14] In order to calculate ex-ante IR for active management we use the variant suggested by Ye (2008) as it excludes influence of the strategy risk which is not an object of this study

Panel A. IC						
Raw IC	Raw IC	Undervalued group	Overvalued group	Large-cap group	Medium-cap group	Small-cap group
Cash ratio	0.001	0.006	0.001	-0.015	-0.019	0.016
Current ratio	0.000	0.009	-0.010	-0.021	-0.020	0.005
Gross Margin	0.005	0.000	0.024	-0.006	0.013	0.024
Operating Margin	-0.002	-0.015	0.013	0.019	0.027	-0.009
Net Margin	0.006	0.007	0.035	0.027	0.039	0.010
D/E	0.004	-0.010	0.013	0.014	0.012	0.014
ROE	0.051	0.079	0.017	0.058	0.087	0.064
ROA	0.041	0.076	0.008	0.050	0.065	0.058
Panel B. Ex-ante IR						
	Raw IC	Undervalued group	Overvalued group	Large-cap group	Medium-cap group	Small-cap group
Cash ratio	0.01	0.07	0.01	-0.14	-0.18	0.22
Current ratio	0.01	0.13	-0.14	-0.19	-0.22	0.06
Gross Margin	0.07	0.01	0.33	-0.05	0.14	0.27
Operating Margin	-0.02	-0.17	0.13	0.16	0.30	-0.07
Net Margin	0.08	0.07	0.35	0.26	0.40	0.07
D/E	0.05	-0.10	0.13	0.10	0.17	0.12
ROE	0.44	0.56	0.13	0.36	0.64	0.54
ROA	0.34	0.51	0.07	0.29	0.50	0.45

Table A2. Ex-ante IC and IR for all stocks and for stocks in contextual groups

Appendix 3

Benefits of applying different factors within certain contexts is also evident when tested with real returns. Portfolios are constructed based on the factor within each context as well as in the general case, then performance of these portfolios is recorded and shown in table A3. The results clearly indicate that when factors are applied in the appropriate context there is significant improvement of performance. For example Gross Margin factor gives 4.7% when constructed on all stocks, but when only Undervalued stocks are used the return improves to 14.2%, while in the Size context Medium company deliver 27% from this factor.

Panel A. Realized Annual Returns						
Factors	Value Context		Size Context			General Portfolio
	Undervalued portfolio	Overvalued portfolio	Large-cap portfolio	Medium-cap portfolio	Small-cap portfolio	
Cash ratio	-4.9%	-2.8%	-3.0%	20.7%	0.1%	-4.2%
Current ratio	-2.3%	4.0%	-4.1%	17.1%	0.5%	2.0%
D/E	6.2%	-2.3%	9.4%	31.4%	1.3%	3.4%
Gross margin	14.2%	-0.4%	-3.0%	27.0%	0.0%	4.7%
Operating margin	30.9%	0.6%	3.8%	42.6%	0.6%	13.5%
Net margin	29.8%	9.2%	13.4%	54.0%	1.3%	20.8%
ROE	40.3%	1.0%	24.1%	62.8%	0.7%	32.2%
ROA	37.1%	-0.4%	14.2%	46.1%	0.3%	20.9%
Panel B. Realized Information Ratio						
Factors	Value Context		Size Context			General Portfolio
	Undervalued	Overvalued	Large	Medium	Small	
Cash ratio	(0.30)	(0.15)	(0.16)	0.87	0.05	(0.22)
Current ratio	(0.20)	0.38	(0.27)	0.86	0.25	0.13
D/E	0.57	(0.22)	0.53	2.18	0.89	0.25
Gross margin	1.36	(0.03)	(0.16)	1.60	0.01	0.35
Operating margin	1.94	0.05	0.22	3.35	0.57	0.84
Net margin	1.72	0.76	0.95	4.43	0.72	1.08
ROE	2.07	0.07	1.30	4.47	0.51	1.57
ROA	2.11	(0.03)	0.72	3.04	0.22	1.11

Table A3. Realized returns and ex-post IR for contextual portfolios compared

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