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HOUSING PRICE DIFFERENCE AND REGIONAL HUMAN CAPITAL MOBILITY—AN ANALYSIS BASED ON GENERAL EQUILIBRIUM MODEL FOR YANGTZE RIVER DELTA REGION OF CHINA

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Abstract

This paper is to provide a preliminary result on the possibility of achieving socially efficient outcome given the complex relationship between housing price difference and human capital mobility within a region across different cities. With a general equilibrium model by slightly modifying the existing work with the feature of *Hukou* included, we find that, if there is there is no congestion in commuting and no spillover in production, the equilibrium allocation can be efficient even with housing price difference and household registration management restriction (*Hukou*) given the existence of equilibrium.

Keywords

Housing Price Difference; Human Capital; General Equilibrium; Social Efficiency; *Hukou*

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1. Introduction

The Yangtze River Delta (YRD) region is an important meeting point of the "One Belt, One Road" and the Yangtze River Economic Belt, and has an important strategic position in China's modernization and all-round opening-up pattern. In order to implement the government's important instructions on promoting the integrated development of the Yangtze River Delta with higher quality, Shanghai and the three provinces of Jiangsu, Zhejiang and Anhui are working together to build a world-class city cluster. Promoting the optimal allocation of talent resources among the Yangtze River Delta city cluster and the inter-regional spillover of innovation power will help promote the higher quality integration of the Yangtze River Delta and more effectively build the Yangtze River Delta world-class city cluster, and will also help implement the 19th National Congress' proposal of breaking down the shortcomings of institutional mechanisms that negatively impact the social mobility of labor and talent, so that everyone can have the opportunity to realize their own development through hard work. However, based on existing studies, it is inferred that the difference in housing prices between cities in the Yangtze River Delta and the high housing prices in major cities may be becoming a barrier to human capital mobility.

Indeed, as an important engine of China's economic development, the Yangtze River Delta region's rapidly rising housing prices have attracted much attention in recent years. According to the latest data, from 2008 to 2021, the average annual increase in house prices in the Yangtze River Delta region is about 10%. For example, the house price in Shanghai rose from an average of about 18,000 yuan per square meter in 2008 to about 60,000 yuan per square meter in 2021; the house price in Hangzhou rose from an average of about 15,000 yuan per square meter in 2008 to about 50,000 yuan per square meter in 2021; the house price in Nanjing rose from an average of about 12,000 yuan per square meter in 2008 to about 40,000 yuan per square meter in 2021 million yuan. High-priced areas tend to be first-tier or core cities in the Yangtze River Delta region, which have more job opportunities, higher salaries, richer public resources, greater wealth accumulation possibilities, and superior education and health care levels. These factors attract a large number of workers to these cities.

However, workers have to face the reality of housing issues when choosing a city of employment. This has a significant impact on the choice of cities of employment and the decision to stay or go to work for highly skilled workers.

High housing prices have a double impact on human capital mobility in the Yangtze River Delta region. On the one hand, high housing prices increase the cost of living, especially for young professionals and entrepreneurs. Against the backdrop of continuously rising housing prices, workers' housing costs increase much faster than their wages. They may face the dilemma of overburdened rent or high threshold for home ownership, and thus choose to move their human capital to regions with relatively lower housing prices. This may have reduced human capital inflows to the YRD region. On the other hand, high housing prices also provide opportunities for investment returns. Those individuals who already own property in the YRD region can grow their wealth through asset appreciation and rental income, which in turn enhances the region's human capital. High housing prices may encourage individuals to concentrate their capital in the real estate market rather than investing in other areas. This may reduce the opportunities for human capital mobility in other sectors and negatively affect the industrial structure and innovation capacity of the YRD region.

As can be seen, while housing prices are rising rapidly in all major cities in the Yangtze River Delta, there are also significant urban differences between housing prices, which in turn lead to differences in talent mobility. High-priced areas attract a portion of high-skilled laborers to seek more employment opportunities and higher salaries. However, due to the fierce competition in first-tier cities in high-cost areas, high-skilled workers tend to have access to better job and exchange opportunities and thus higher expected income. Meanwhile, although housing prices are relatively low in small and medium-sized cities, they lack an external environment suitable for innovation and entrepreneurship of highly qualified workers due to their relatively low level of economic development and agglomeration. Therefore, even with lower house prices, migration to small and medium-sized cities may lead to a faster decline in nominal income and possibly lower real income. This implies that the crowding-out effect of high housing prices on low-skilled labor may be more pronounced, thus indirectly leading to the optimization of the human capital structure in the Yangtze River Delta region.

For example, as the core city and economic center of the Yangtze River Delta region, Shanghai has higher housing prices. This leads to some high-skilled laborers having higher expected income and being able to afford the cost of living due to high housing prices. At the same time, Shanghai has better educational and medical resources, as well as rich cultural and business opportunities, which attracts a large number of highly qualified people to Shanghai. High housing prices screen out the flow of talent to a certain extent, and only those with certain economic strength or high skills can stay in Shanghai. Hangzhou, on the other hand, has relatively low housing prices compared to Shanghai, but is still a relatively high-priced city. Hangzhou has a good environment for entrepreneurship and innovation, which attracts a large number of entrepreneurs and skilled talents. Despite the high housing prices, Hangzhou offers abundant job opportunities and development prospects, making the highly skilled labor force willing to bear the burden of high housing prices. Cities with high housing prices attract talented people with strong economic power or high skills who can afford the cost of living and seek better employment and development opportunities in the city. This phenomenon is known as "talent retention". Conversely, cities with low housing prices may be attractive to lower- and middle-income people or other workers with relatively low human capital endowments, but less attractive to high-skilled workers.

The difference in house prices among cities in the Yangtze River Delta region shapes the pattern of human capital mobility to some extent. High-priced cities attract high-skilled labor and enhance innovation and entrepreneurship and technological development; low-priced cities attract low- and middle-income people but may lack the advantage of attracting high-skilled labor. This pattern of human capital flow has important implications for economic development and industrial upgrading in the Yangtze River Delta region, and requires policy formulation to take into account the house price factor and balance the human capital flow between different cities to achieve sustainable regional economic development.

Also, there are obvious differences in the industrial layout of the cities in the Yangtze River Delta region. Shanghai, as the economic center and international financial center of the Yangtze River Delta region, is dominated by modern service industries such as finance, commerce, shipping, and technology innovation, and has many multinational companies and large domestic corporate headquarters. Nanjing, as the capital city of Jiangsu Province, is dominated by manufacturing and modern service industries, covering a wide range of fields such as automotive, electronics, aerospace, and finance. Suzhou is dominated by high-tech industries and manufacturing, and is especially good at electronic information, new materials, biomedicine and other fields. Hangzhou, on the other hand, has the internet and digital economy as its core, focusing on high-tech industries such as e-commerce, cloud computing, and artificial intelligence.

The differences in the industrial layout of these cities have an important impact on human capital flows. Highly skilled workers tend to prefer cities that are relevant to their professions and skills. Shanghai, as a financial and trade center, attracts a large number of financial, business and innovative talents. Nanjing's manufacturing base and technological innovation capability attracts a number of engineers and manufacturing professionals. Suzhou's high-tech industry and innovative entrepreneurial atmosphere attracts professionals in the technology sector.

Hangzhou, a major city in the Internet and digital economy, attracts many talents in Internet, software development and data analysis and other related fields.

Given all these complex backgrounds, in order to analyze the problem of housing pricing difference and regional human capital mobility and their welfare consequence, we need to rely on the general equilibrium framework. In this paper, we provide some benchmark analysis within the general equilibrium framework to answer two essential questions: by modelling all these different aspects into the general equilibrium framework, whether the efficiency results can be established? If it can be established, then what are the conditions?

The answers would help us to understand whether the government should put more effort on planning the development of the region as a whole and how the government may possibly do if some conditions are violated.

The paper is organized as follows. In Section 2, we review some relevant literature. In Section 3, we provide the model. In Section 4, we show the main result. Section 5 concludes the paper.

2. Literature Review

House prices are widely considered to be an important factor influencing human capital mobility. This literature covers findings from multiple countries and regions on the impact of house prices on human capital mobility. Among them, some studies suggest that rising house prices increase the cost of living for labor and reduce disposable income, thus negatively affecting human capital accumulation and prompting labor mobility to other regions. Other studies point out that rising house prices negatively affect business location conditions, overall income and welfare, and inhibit regional labor aggregation and free mobility.

Helpman (1998) included housing prices in the study of labor mobility as a substitute for the cost of agricultural products and applied it to the classical "center-periphery" model. Hanson (2005) and others find that house prices will affect the regional concentration of labor, and that rising house prices will cause labor diversion. However, Dohmen (2005) argues that the expectation of house price arbitrage will drive labor inflows. Maclennan et al. (1998) finds that rising house prices increase the cost of living, reduce disposable income and living standards, and negatively affect labor pooling, driving labor to other regions, which would have a disincentive effect on labor aggregation and impede the free movement of labor between regions. Rabe and Taylor (2012) find that housing price differences across cities are a key factor in the collective migration of labor households using survey data from 1992 to 2008 in the U.K. Plantinga et al. (2013) finds, based on data on labor migration intentions in the U.S., that intended city rising housing prices reduce labor force selectivity to the region.

Gao et al. (2012) find that higher relative house prices in cities induce labor outflows, while the disincentive effect of house prices is more reflected in rural labor. Fan et al. (2015) argue that high urban house prices do not inhibit the continued inflow of foreigners because the majority of the new resident population is low-skilled labor, who are not closely related to residential transactions. Xia and Lu (2015) find that house prices "capitalize" some unobserved public services or urban characteristics, thus making house prices also have a positive effect on labor inflow. Zhang et al. (2017) find an inverted U-shaped relationship between house prices and labor mobility, with a relatively smaller inverted U-shaped inflection point for high-skilled labor and a larger inverted U-shaped inflection point for labor mobility in coastal cities. Chen et al. (2019) shows that Chinese mega-cities experience a high housing price due to crowding out of the elite population, but overall, elites still prefer supercities. Yang and Pan (2020), using panel data from 31 Chinese provinces and cities, finds that in the long run, human capital accumulation can promote regional economic development and lead to higher house prices; at the same time, higher house prices can hinder economic development and thus affect human capital accumulation. Zhou and Li (2020) found that house prices and individual human capital investment showed a nonlinear first-promoted-then-suppressed relationship, and that economic development of cities had a positive effect on the accumulation of human capital.

Human capital is naturally related to knowledge spillover. Henderson (1974) earlier discussed the issue of residents' choice of regions under the exogenous given knowledge spillover. Martin et al. (1999) found that knowledge spillover has spatial decay and time lag effect, which will lead to the difficulty of effective diffusion and transfer of innovation results and trigger the non-convergence results of inter-regional economic growth in the process of innovation-driven endogenous economic growth. Fujita et al. (2004) argue that knowledge spillovers, as one of the important ways of spatial interaction, will lead to regional convergence due to their unique spatial decay and time lag effects. Berliant and Fujita (2008) study resident learning behavior and spillovers within a single region, and Allen and Arkolakis (2014) find that spillovers have a significant effect on the regional distribution of economic activity in equilibrium, based on simple assumptions about the potential dependence of productivity and population. Allen, Arkolakis and Li (2015, hereafter simplified as AAL) construct the first quantitative general equilibrium regional model with trade in goods, commuting, and knowledge spillovers, and then explore the issue of optimal urban structure. Diamond (2016) empirically investigates the impact of human capital and knowledge spillovers on regional prices, productivity, and income inequality. Davis and Dingel (2019) first explored the reasons why knowledge spillovers are mostly in large cities based on a city-region model with knowledge communication costs. Wan et al. (2010) found that there is a direct correspondence between the degree of spatial agglomeration of innovation and the level of regional economic development in China. Chen and Liang (2014) introduced the hypothesis of heterogeneous labor and knowledge spillover and analyzed the heterogeneous labor location choice and the resulting urban equilibrium structure in the urbanization process of transition economies. Zhang and Qiao (2016) found that the knowledge spillover effect among neighboring provinces is conducive to reducing the regional income gap. Ye et al. (2018) found that stronger knowledge spillover effects are an important motivation for cities to attract mobile populations, especially entrepreneurial individuals, and in turn enhance regional entrepreneurial activity.

The impact of house prices on labor mobility is a complex process. On the one hand, the increase in house prices increases the cost of living of labor force, which has a negative push effect on labor force inflow. On the other hand, high-price cities usually have better infrastructure, living conditions, employment opportunities and room for wealth growth, which are attractive to labor force. However, high housing prices may also lead to labor outflow and industrial relocation, particularly affecting low-skilled labor and students from weak family backgrounds. In terms of labor force heterogeneity, high-skilled labor force is more sensitive to house prices and has stronger demand for home ownership. Domestic and international studies on the relationship between labor and house price volatility show that the correlation between the two is not only influenced by other economic factors, especially labor heterogeneity. And as Fajgelbaum et al. (2020) reveal, knowledge spillovers from a heterogeneous labor force will have a significant impact on regional development. On the other hand, the spillover effects of regional agglomeration will affect economic factors such as population flows and interregional prices (Monte et al., 2018). These changes are rarely studied in a systematic framework in the domestic literature, probably due to the fact that knowledge spillovers are more often analyzed from an empirical perspective. Although the foreign literature on quantitative regional economics mostly uses general equilibrium models with rich characteristics of economic activities, it cannot describe the specificity of domestic property market and household registration management in China. This paper is an attempt to extend the existing quantitative regional economics models to the Yangtze River Delta region to enrich the research in China.

3. Model

This model is mainly based on AAL. We extend some of its setting to better explain the observed facts of YRD region. There is a region, say, for example, YRD Region, consisting a number of different cities (we sometimes name it as “places” in the following context), the set of which is denoted as $S = \{1, 2, ... ,N\}$. The commercial relations among these cities are affected by different factors in the real world. In our model, following AAL, we assume they are impacted by two different kinds of costs. The first one is widely defined in literature, that is iceberg trade cost. To be specific, it means, if we want to ship one unit of product from $i \in S$ to $j \in S$, then we must ship $\tau_{ij} \geq 1$ units. For travel cost, it means it takes t_{ij} units of time for the worker to travel from $i \in S$ to $j \in S$.

We model the real estate market. Following AAL, we assume there are two kinds of buildings, one is for residential use, particularly for workers, the supply of which is denoted as $H_j^R: S \to R$, and another is for firms' commercial use, the supply of which is $H_j^F: S \to R$. Following AAL, a general form of technology for the firm is considered and we put on specific assumption on the consumer utility function. As standard in literature, perfectly competitive goods markets are assumed.

Now, we describe the firms. The set of all firms in this economy is denoted as Δ. Each element δ of this set is an individual firm. Firms can produce different kinds of products. We use Θ to denote the set of all products. In addition, each firm δ can product its firm-specific product, which is denoted as $\vartheta(\delta) \in \Theta$. In this model, we don't consider the case that one firm producing in multiple cities. We use $K(\delta) \in S$ to denote the city where firm δ operates.

There can be more than one firms in one city. For city $j \in S$, we denote the set of firms there as $\Delta_j = \{\delta | K(\delta) = j\}$. There can be more than one firms producing the same product $\theta \in \Theta$. $\Delta_\theta = \{\delta | \vartheta(\delta) = \theta\}$ includes all the firms producing the same product θ.

We assume firm would mainly rely on effective workers and plants for production, where the requirement for plants induces the demand for buildings. We can therefore define a general product function $y_\delta = f_\delta(l, h)$ for firm δ. Here, l is the number of effective workers employed by the firms with a local wage $w_{K(\delta)}: S \to R_{++}$. h is the amount of commercial use buildings the firm need to use with a rent $r_{K(\delta)}^F: S \to R_{++}$.

Now, we can define each firm's profit maximization profit as follows:

$$\max_{l,h} \pi_\delta = p_\delta f_\delta(l, h) - w_{K(\delta)} l - r_{K(\delta)}^F h \qquad (1)$$

Ω is to denote the set of all workers, where each worker is denoted as $\omega \in \Omega$. These workers can live and work in different cities. In the context of YRD region, we assume that the place for living is the workers' permanent living place, while the place for working can be workers' working place. For living, each worker needs to rent the residential building and buy some products.

Each worker ω has some initial endowment of time $e(\omega)$: $\Omega \rightarrow R_{+}$. The usage of time is divided to be four: for commuting between living and working place, denoted as $e^{c}(\omega)$; for social activity intending to improve productivity, denoted as $e^{A}(\omega)$, for working firms, denoted as $e^{w}(\omega)$, and for leisure, denoted as $e^{l}(\omega)$. An worker can choose its permanent living place $i \in S$ and its working place $j \in S$. Travelling across different place will take its time, which can be denoted as $e^{c}(\omega) = t_{ij}$.

The income of a worker includes two parts: wage and rent. The worker's wage depends on its productivity level (defined as $A_{\omega}(e^{A}(\omega), i, j)$.), the wage level in its working place j (defined as w_j) and the time the worker spends on working, which induces the wage income as $w_j A_{\omega} e^{w}(\omega)$. Notice that if the worker chooses to work in some places too far away from the working place, the worker would have limited time for working. However, the rental income may make up the loss in wage income.

All the workers together own the residential and commercial buildings and also own parts of the firms' profits. These contribute to workers' rental income. $\{s_{\delta}(\omega)\}_{\delta \in \Delta}$ is to denote worker ω's the stock share in firm δ. $\{s_k^R(\omega), s_k^F(\omega)\}_{k \in S}$ is to denote worker ω's shares on residential and commercial buildings in each place k. In the context of YRD region, or other cities with high real estate market returns, whether the worker can get the real estate in one city should depend on the requirement of household registration management, or *Hukou* in China. Accordingly, we assume the worker can surely get the shares on residential buildings in his permanent living place i, while the probability of getting shares in other places is a function of the local wage level, denoted as $\zeta(w_k) \in (0,1]$. $\zeta(w_k)$ is decreasing in w_k and equal to 1 if $k=i$. In YRD region, the restriction on purchasing residential buildings is usually placed in well-developed cities such as Shanghai. Even though it is not a perfect match, the local wage in these cities is usually larger than other cities averagely. Accordingly, we consider our assumption captures the real world in a good way.

We can define the utility for a worker ω who lives in $i \in S$ and works in $j \in S$ as follows:

$$\boldsymbol{u_{\omega}(e^{l}(\omega), \{g_{\theta}(\omega)\}_{\theta \in \Theta}, h_i^R(\omega), i, j)} \qquad \textbf{(2)}$$

Here, each worker ω consumes a variety of final products as the set $\{g_{\theta}(\omega)\}_{\theta \in \Theta}$. Also, each worker ω needs to use some residential building (measured in units of area), the amount is denoted as $h_i^R(\omega)$. The quantity of good $\theta \in \Theta$ that the worker ω consumes is:

$$\boldsymbol{g_{\theta}(\omega) = \sum_{\delta \in \Delta_{\theta}} \frac{q_{\delta}(\omega)}{\tau_{K(\delta)i}}} \qquad \textbf{(3)}$$

$\tau_{K(\delta)i}$ is the iceberg cost of shipping products from city $K(\delta)$ to i; $q_{\delta}(\omega)$ is the amount of products worker ω purchased from the producer δ.

Naturally, we assume workers should welcome more leisure, more products consumed and larger houses. Accordingly, the utility function of each worker ω is increasing $e^{l}(\omega)$, $\{g_{\theta}(\omega)\}_{\theta \in \Theta}$, and $h_i^R(\omega)$.

The optimization problem for each worker ω is as follows:

$$\boldsymbol{\max_{q_{\delta}(\omega), h_i^R(\omega), \{e^{A}(\omega), e^{w}(\omega), e^{l}(\omega)\}, i, j} u_{\omega}(e^{l}(\omega), \{g_{\theta}(\omega)\}_{\theta \in \Theta}, h_i^R(\omega), i, j)}$$

subject to their budget constraint:

$$\boldsymbol{\sum_{\delta \in \Delta} p_{\delta} q_{\delta}(\omega) + r_i^R h_i^R(\omega) \leq \sum_{\delta \in \Delta} s_{\delta}(\omega)\pi_{\delta} + \sum_{k \in S} \zeta(w_k) r_k^R s_k^R(\omega) + \sum_{i \in S} r_i^F s_i^F(\omega) + w_j A_{\omega} e^{w}(\omega),} \qquad \textbf{(4)}$$

and their time constraint:

$$\boldsymbol{e^{c}(\omega) + e^{A}(\omega) + e^{w}(\omega) + e^{l}(\omega) \leq e(\omega),} \quad (5)$$

where $A_{\omega} = A_{\omega}(e^{A}(\omega), i, j)$ is the effective units of labor the worker provides.

As each worker ω needs to optimize its utility by choosing the places for living and working, we can denote therefore denote the choices as functions $I(\omega)$ and $J(\omega)$. As to city i for living, the set of all workers living there is denoted as $\Omega^{I}_{i} = \{\omega | I(\omega) = i\}$. Similarly, we define $\Omega^{J}_{j} = \{\omega | J(\omega) = j\}$ as the set of all workers working in city j. $L_j^E = \int_{\omega \in \Omega_j^J} A_{\omega} e^{w}(\omega) d\omega$ is to define the effective labor in city j.

4. Main Result

Following AAL, we can define the equilibrium for the slightly modified model. In equilibrium, the markets for labor, goods, residential and commercial buildings markets must be cleared and we have the following conditions:

$$\int_{\omega\in\Omega_j^J} A_\omega e^w(\omega)d\omega = \sum_{\delta\in\Delta_j} l(\delta)$$

$$\int_{\omega\in\Omega} q_\delta(\omega)d\omega = f_\delta(l(\delta),h(\delta))$$

$$\int_{\omega\in\Omega} h_i^R(\omega)d\omega = H_j^R$$

$$\sum_{\delta\in\Delta_j} h(\delta) = H_j^F$$

We then give the modified definitions based on AAL.

Definition 1 (as in AAL).
Given each worker ω's endowment $\{e(\omega),\{s_\delta(\omega)\}_{\delta\in\Delta},\{\zeta(w_k)s_k^R(\omega),s_k^F(\omega)\}_{k\in S}\}$, production function $f_\delta(l,h)$ for each firm $\delta\in\Delta$ and building supply $\{H_k^R,H_k^F\}_{k\in S}$, the sets of worker ω's consumption choices $\{\{q_\delta(\omega)\}_{\delta\in\Delta},\{h_k^R(\omega)\}_{k\in S},\{e^A(\omega),e^w(\omega),e^l(\omega)\},I(\omega),J(\omega)\},\{l(\delta),h(\delta)\}$ and the price variables $\{\{r_k^R,r_k^F,w_i\}_{k\in S},\{p_\delta\}_{\delta\in\Delta}\}$ constitutes **a spatial equilibrium with zoning,** equilibrium $\mathcal{F}_1$, if (i) equations (6)-(9) are satisfied; (ii) workers and firms maximize the utility and profit as above.

Definition 2 (as in AAL).
The above equilibrium $\mathcal{F}_1$, and $r_i^R = r_i^F$ and $H_j^R + H_j^F = H_j$ constitutes a **spatial equilibrium without zoning** $\mathcal{F}_2$.

The following is the main result which provides sufficient conditions for establishing the efficiency of the spatial equilibrium without zoning.

Theorem 1 (as in AAL).
In the spatial equilibrium $\mathcal{F}_2$, assuming no congestion in commuting and no spillover in production so that $t_{ij}(\omega)$ and $A_\omega(e^A(\omega),i,j)$ only depends on ω's individual choice $(e^A(\omega),i,j)$, then if an equilibrium exists, it is efficient.
Proof. We mainly follow AAL to conduct the proof for the slightly modified model. AAL's strategy is to design a setting equivalent to the model but easier to apply the first fundamental theorem of welfare economics.

As to the initial endowments of each worker, the time needs to be modified. Here we assume each worker ω has time endowment $e_{ij}(\omega) = e(\omega) - t_{ij}$ for each commuting pair (i, j). This $e_{ij}(\omega)$ can be divided into three parts: $e^A_{ij}(\omega)$ for increasing productivity, $e^w_{ij}(\omega)$ for work, and $e^l_{ij}(\omega)$ for leisure.

The largest effective labor can be defined as

$$\boldsymbol{e_{ij}^e(\omega,t) = \max_{e_{ij}^A+e_{ij}^w\le t} A_\omega\left(e_{ij}^A(\omega),i,j\right)e_{ij}^w.}$$

The effective labor endowment is denoted as $\tilde{e}_{ij}(\omega) = \max_{t\le e_{ij}(\omega)} e_{ij}(\omega,t)$. e_{ij}^l is to denote modified effective leisure. Accordingly, we can define the real leisure as

$$\boldsymbol{l^{real}\left(\omega,e_{ij}^l\right) = \max\{t|\tilde{e}_{ij}(\omega) - e_{ij}^l - e_{ij}^e\left(\omega,e_{ij}(\omega)-t\right)\ge 0\}.}$$

The utility function is modified so that each worker's utility would be measured based the consumption and leisure plan $z(\omega)$ denoted as

$$\boldsymbol{\left\{\{g_{\theta k}(\omega)\}_{\theta\in\Theta,k\in S},\left\{h_k^R(\omega)\right\}_{k\in S},\{e_{ij}^l\}_{i,j\in S}\right\}.}$$

Here, $\{g_{\theta k}(\omega)\}_{\theta\in\Theta}$ is the products consumed by worker ω in city k. We also define $u_\omega\left(e_{ij}^l(\omega),\{g_{\theta i}(\omega)\}_{\theta\in\Theta},h_i^R(\omega),i,j\right) = -\infty$ if $h_i^R(\omega) = 0$ or for all $\theta\in\Theta$ $g_{\theta i}(\omega) = 0$.

Given the consumption and leisure plan, worker ω's utility is

$$\boldsymbol{U(\omega) = \prod_{(k,l)\neq(I_{max}(z(\omega)),J_{max}(z(\omega)))} 1_{l^{real}(e_{mn}^l)\ge e_{mn}(\omega)} \max_{i,j} u_\omega\left(l^{real}(\omega,e_{ij}^l),\{g_{\theta i}(\omega)\}_{\theta\in\Theta},h_i^R(\omega),i,j\right)}$$

where

$\boldsymbol{\left(I_{max}(z(\omega)),J_{max}(z(\omega))\right) = arg\max_{i,j} u_\omega\left(l^{real}(e_{ij}^l),\{g_{\theta i}(\omega)\}_{\theta\in\Theta},h_i^R(\omega),i,j\right)}$, $\boldsymbol{g_{\theta i}(\omega) = \sum_{\delta\in\Delta_\theta}\frac{q_{\delta i}(\omega)}{d_{K(\delta)i}}}$ is the products the worker ω consumes at city I where $q_{\delta i}(\omega)$ satisfies the constraint $\sum_{i\in S} q_{\delta i}(\omega)\le q_\delta(\omega)$.

The worker needs to make decisions based on the following modified problem

$$\max_{\{q_{\delta k}(\omega)\}_{\delta\in\Delta,k\in S},\{h_i^R(\omega)\}_{i\in S},e_{i,j}^l} U(\omega)$$

such that

$$\sum_{\delta\in\Delta} p_\delta q_\delta(\omega) + \sum_{k\in S} r_k^R h_k^R(\omega) + \sum_{m,n\in S} w_n e_{mn}^l \leq \sum_{\delta\in\Delta} s_\delta(\omega)\pi_\delta + \sum_{k\in S} r_k^R[\zeta(w_k)s_k^R(\omega) + s_k^F(\omega)] + \sum_{m,n\in S} w_n \tilde{e}_{mn}(\omega)$$

where $q_{\delta k}(\omega)$ is the amount of products worker ω purchased from firm δ shipping to city k. It can be verified as in AAL that the worker in this modified environment makes decisions exactly the same as in the model setting. The behavior of firms and the market clearing conditions are not modified here.

Define the allocation $\{\{x_\omega\},\{y_\delta\}\}$ of the economy. There are three cells in both x_ω and y_δ: the first cell is for the goods quantity and firm share; the second is for the shares of buildings; the third is for the effective time. Specifically,

$$x_\omega = \left\{\{(q_\delta(\omega),0)\}_{\delta\in\Delta}, \left\{h_k^R(\omega)\right\}_{k\in S}, \{e_{mn}^l(\omega)\}_{m,n\in S}\right\}$$

and

$$y_\delta = \left\{\{(0,0),\dots,(0,0),\dots,(Y_\delta,-1),(0,0),\dots,(0,0)\},\{0,\dots,0,-h(\delta),0\},\{0,\dots,0\}\right\}.$$

The endowment of the whole economy is

$$e = \left\{\{0,0\}_{\delta\in\Delta}, \{H_i\}_{i\in S}, \{\textstyle\sum_{k\in S}\int_{\omega\in\Omega} e_{kl}^e(\omega)d\omega\}\right\}.$$

The corresponding price is $p = \{\{p_\delta,\pi_\delta\}_{\delta\in\Delta},\{r_k^R\}_{k\in S},\{w_k\}_{k\in S}\}$.

Given the condition in the theorem, the local non-satiation assumption of original utility preferences would induce the local non-satiation property of the utility we have modified. Therefore, first fundamental theorem of welfare economics can be applied and we are done with the proof.

5. Conclusion

By slightly modifying the settings in AAL, we can build a model to analyze the housing price difference and regional human capital mobility with the feature of household registration management, or *Hukou*, being considered. With this model we have established a preliminary efficiency result for an economy with housing price difference and regional human capital flow. We find that, if workers' commuting time is not affected by the others and there is no production spillover, then the equilibrium allocation can be efficient given the existence. In this case, even though the housing price can be different across cities within the region, the human capital flow would achieve the social optimal outcome. Particularly, compared to AAL's findings, our results indicate that, if the right for *Hukou* is highly related to the economic development level of the city, then *Hukou* would not increase the difficulty of achieving the efficiency outcome under the same conditions.

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