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Partner Trust Level in Collaborative Demand Forecast Sharing and its Impact on Supply Chain Profitability

Seong-Hyun Nam [1]

[1] *Professor of Operations & Supply Chain Management, Middleton School of Entrepreneurship & Management, Nistler College of Business and Public Administration, University of North Dakota, USA*

Abstract

Considering trust as a critical factor for successful collaborative demand forecast sharing, this paper studies the dynamic relationship between trust level-based forecasting capability and supply chain profitability. We develop a forecasting coordination model to examine how the enhanced partner trust level can influence the forecasting evolution to improve demand forecasts' accuracy. We estimate costs and profits from demand forecast sharing under varying trust levels between a buyer and a partner supplier and then determine the optimal level of trust for both parties to create the maximum economic value through collaborative demand forecast sharing. To assess the opportunity costs associated with no demand forecast sharing, we compare a joint forecasting supply chain's profitability with a supply chain where the supply chain partners maintain separate demand forecasts. We find that once the buyer and the supplier agree to engage in demand forecast sharing with a joint goal of supply chain profit maximization, they should be able to retain the collaborative trust that is as closer to an absolute level as possible throughout their working relationship. Thus, the model presented in this study may help both the buyer and the partner supplier evaluate how supply chain profitability can improve as they modify their partner trust levels and determine the optimum trust-level policy for mutual benefits.

Keywords: Collaborate demand forecast sharing, Demand forecasting evolution, An optimal level of trust, Supply chain forecasting coordination, Supply chain profitability.

1. INTRODUCTION

One of the strategic initiatives for dealing with demand uncertainty and supply risk in today's turbulent marketplace is facilitating process integration with collaborative demand forecast sharing across all involved supply chain partners. By incorporating the collective knowledge, skills, and abilities of buyers and suppliers into a coordinated joint forecasting system, the trading partners can make more accurate and timely market demand predictions and improve their organizational and business performance (Dyer and Chu, 2003; Terwiesch et al., 2005; Trapero et al., 2012; Eksoz et al., 2014). Successfully executing the collaborative forecasting initiatives can also help them build mutual trust, promoting synergetic improvement, competitive performance, and an innovative working relationship among supply chain partners (Cheng et al., 2008; Fawcett et al., 2012).

However, various issues and problems make it risky and challenging for supply chain partners to engage in demand forecasting collaboration. First, although buyers' and suppliers' forecasted demands are handled as private information, both sides may have to continuously exchange them during the collaborative forecasting process to make optimum operational decisions. To achieve effective joint forecasting performance and avoid unnecessary costs, supply chain partners need to share confidential forecast data, including inventory levels, available production capacity, and product/market strategy (Kwon & Suh, 2004; Kwon et al., 2012; Ali et al., 2012; Gao, 2015). Second, greater interdependence requiring the exchange of sensitive information with partners can increase exposure to vulnerability (e.g., Fawcett et al., 2007; Poppo et al., 2008). Therefore, without the presence of mutual trust and the willingness to take a risk, buyers and suppliers may be reluctant to commit to sharing private information. Third, buyers and suppliers in the supply chain may hold unequal trust levels toward their counter partners. The level of trust in the buyer-supplier relationship is not the same in all situations. The discrepancy in trust level between partners may not motivate them to see value creation potential in increased commitment to sensitive information sharing. Considering these issues, many scholars have studied the nature and dimensions of supply chain trust and its impact on forecasting collaboration. They have also examined the benefits of forecasting partnerships in supply chain coordination and suggested that a successful buyer-supplier relationship in forecast information sharing depends on relational forms of exchange represented by a higher level of partner trust. However, there is a lack of research to explain (1) how the enhanced partner trust level can influence the forecasting evolution process to improve demand forecasts' accuracy and (2) how to determine an optimal level of partner trust that can promote mutually profitable and cost-effective outcomes for buyers and suppliers. Enhancing the trusting relationship in collaborative demand forecast sharing may come at a cost as it may require investments for effective interaction mechanisms in technology, communication networks and facilities, and additional employee training (Sherman, 1998; McCarthy & Golicic, 2002). To avoid over-investment (or under-investment) and unnecessary risk and to select the best trust-level policy, supply chain partners must have the ability to assess how the costs and expected profits from the demand forecast sharing can be changed as they adopt different levels of partner trust. In this study, focusing on the questions mentioned above, we explore the dynamic relationship between partner trust level, forecasting capability, and supply chain profitability and investigate how the three factors can affect a demand forecast sharing decision.

The key research objectives of this paper are: to examine the synergy effects of the combined forecasting capability of a supply chain on supply chain profitability; to estimate forecasting costs and profits from collaborative demand forecast sharing based on varying trust levels between a buyer and a partner supplier; to determine the optimal level of trust for both parties that can create the maximum economic value through collaborative demand forecast sharing. To assess the opportunity costs associated with no demand forecast sharing, we compare a joint forecasting supply chain's profitability with a supply chain where the supply chain partners maintain separate demand forecasts (Aviv, 2001). The model developed in this paper will help decision-makers formulate the optimal trust-level policy that best fits their organizational goals and achieves the desired results.

The remainder of this paper is organized as follows. In Section 2, we describe our research background by briefly reviewing the related literature. We develop the trust-level coordination model in Section 3. Section 4 presents results and numerical analysis on the role of partner trust level in collaborative demand forecast sharing and its impact on supply chain profitability. Finally, we summarize our significant findings and discuss managerial implications and future research directions in Section 5.

2. LITERATURE REVIEW

By viewing demand forecast sharing as one of the most critical drivers for almost all supply chain-related decisions, numerous scholars and researchers have studied the forecasting optimization process with supply chain integration. Trapero *et al.* (2012) provide robust empirical evidence that information sharing is a way to accomplish collaboration and improve forecasting accuracy. A study by Babai *et al.* (2013) finds that buyer-supplier collaboration in demand forecasting can improve operational performance. Some other scholars stress the role of supply-side information sharing in improving demand forecasting accuracy and suggest that the supply side needs to be sufficiently agile to act effectively upon the information exchanged via collaborative forecasting (Aviv, 2007; Zhou et al., 2013). Ali *et al.* (2017) confirm that irrespective of the forecasting method adopted, sharing information is always beneficial as the upstream supply chain links use actual consumer demand in their planning framework.

Collaboration requires interaction mechanisms to ensure that the supply chain members make appropriate decisions, resolve unwanted conflicts, and efficiently acquire and use resources. Cachon and Lariviere (2001) point out that some suppliers may not trust the buyer's forecast because of the buyer's incentive to inflate its demand forecast (Durango-Cohen & Yano, 2006; Özer & Wei, 2006; Terwiesch et al., 2005). To deal with this type of problem in demand forecast sharing, they develop contracting models that allow the supply chain to share credible demand forecasts under either forced or voluntary compliance. Numerous studies (Gulati & Nickerson, 2008; Durango-Cohen & Yano, 2011; Özer, Zheng, & Chen, 2011; Ebrahim-Khanjari et al., 2012), however, reveal that contract alone may not be adequate for effective cooperation in demand forecasting and demonstrate how trust is built over time and how collaborative demand forecast sharing can arise without complex contracts. When supply chain partners are honest, the contract will achieve the effectiveness of supply chain coordination in most instances and provide both excellent performance and flexibility in structuring contracts. Also, if trust exists when they enter into an exchange relationship, they may use fewer formal contractual governance methods.

Trust in supply chain relationships is considered a context-dependent, multi-perspective, and multidimensional concept (Klein & Marx, 2018), consisting of various attributes such as integrity, fairness, loyalty, openness, and competence (Riddalls et al., 2002) with multiple types of trust in supply chain relationships such as characteristic trust, rational trust, and institutional trust (Laeequddin et al., 2012; Tejpal el al., 2013). Inter organizational trust in a supply chain is an essential factor in achieving effective and efficient supply chain performance. Trust-based relational mechanisms with information integration, for example, can help supply chain partners to reinforce cooperation and mitigate the risk arising from unanticipated events (Wei et al., 2012; He et al., 2014). Ojha *et al.* (2016) provide evidence that trust can play an essential role in developing entrepreneurial and innovative supply chains. Firms of today are operating under global competition with uncertainty in both demand and supply. To improve demand forecasts' accuracy under such environmental uncertainty and make optimal manufacturing plans, buyers and suppliers should share private information. This activity requires trust between trading partners. Sridharan and Simatupang (2013) highlight that information sharing will only occur if trust exists in the relationship. Sharing confidential information serves as a signal of good faith between supply chain partners because they are willing to make themselves vulnerable. The level of perceived trust in a dependence situation can reduce the impact on perceived vulnerability in business relationships and balance the negative consequences of increased perceived dependence, which may positively impact the level of perceived vulnerability (Svensson, 2004). However, the asymmetry in trust levels between a buyer and a supplier can weaken the positive collaborative behaviors in their interfirm relationship (Thomas & Skinner, 2010; Chiu et al., 2016). The unequal trust levels among supply chain partners may not stimulate them to identify value creation potential in increased commitment to sensitive information sharing.

Although a trust can promote supply chain collaboration, and the advantage resulting from the collaborative partnership can positively affect firms' organizational performance in the supply chain (Uca et al., 2017), building a trust-enabled relationship is difficult and potentially costly, both in needed investments and exposure to vulnerability (Fawcett et al., 2012). Therefore, it is crucial to evaluate the magnitude of how profit increases and cost savings can be achieved through forecast information sharing. Utilizing a quantitative analysis, Dyer and Chu (2003) study the role of trustworthiness in reducing transaction costs and improving performance. On the other hand, Shin and Tunca (2010) show how coordinating contracting plans can solve inefficiency and the loss of supply chain surplus. According to Hyndman et al. (2013), sharing vital information, including private forecasting data, among supply chain participants can reduce transaction costs and minimize uncertainty and risk.

One strategy for buyers to improve the accuracy of demand forecasts and reduce forecasting errors, including disruption risks, is to delay ordering decisions as late as possible to incorporate updated information and data into demand forecasting (Tan, 2002). However, this process may cause friction with suppliers since they are often unwilling to receive buyers' frequent changes or varying updates in order quantities. Therefore, both parties need to create and manage a cooperative working relationship through collaborative demand forecast sharing to resolve these issues. Demand forecast sharing, however, may not be beneficial to supply chain partners unless truthful demand forecasts are exchanged in a timely manner between the buyer and the partner supplier (Matchette & Seikel, 2004; Li & Zhang, 2008; Ren et al., 2010; Durango-Cohen & Yano, 2011; Han & Dong, 2015). Moreover, demand information sharing at the right time depends on the sufficient level of trust between the two parties (Önkal et al., 2008).

This paper focuses on the synergy effect of combined forecasting capability between a buyer and a partner supplier. We study the role of partner trust level in the demand forecast evolution process and address its expected impact on supply chain surplus.

3. MODEL FORMULATION

NOMENCLATURE

$d(t)$ Market demand at time *t*.

$D(t)$ Cumulative market demand by time *t*.

$dF_i(t)$ Demand forecast evolution stochastic processes.

$\sigma_i dW(t)$ Volatility of the forecasting evolution processes.

$F_i(t)$ Supply chain partner *i*'s cumulative forecasted market demand by time *t* (*i=1, 2, 3,4,* & *5*).

$M_i(t)$ Target of supplier *i*'s demand forecast.

$K_i(t)$ Expected value of $F_i(t)$, where $K_i(t) = E[F_i(t)]$.

$G_i(t)$ Expected value of $F_i^2(t)$, where $G_i(t) = E[F_i^2(t)]$.

$Var_i(t)$ Variance of $F_i(t)$, where $Var_i(t) = E[F_i(t) - K_i(t)]^2 = G_i(t) - \{K_i(t)\}^2$.

$L(t)$ Expected loss due to overstock and understock.

$N_i(t)$ Probability that $F_i(t) \leq D(t)$ at time *t*, where $N_i(t) = P[t|F_i(t) \leq D(t)]$.

$r_i(t)$ Supply chain partner *i*'s level of trust in their counterparts at time *t*, where $r_i(t) \in [0,1]$.

$r_{1i}^T(t)$ Buyer's trust level in supplier *i* regarding how truthfully the buyer will be willing to share his forecasted demand information with supplier *i*, where $r_{1i}^T(t) = f\{r_1(t)\}$.

$r_{i1}^T(t)$ Supplier *i*'s trust level in the buyer regarding how supplier *i* will adjust the demand forecast received from the buyer, where $r_{i1}^T(t) = f\{r_i(t), r_{1i}^T(t)\}$.

$C_i^F(t)$ $C_i^F = C^F\{\alpha_i(t)\}$, which are the forecasting expenses for *i =1, 2, 3, 4,* and *5*.

$C_i^{CF}(t)$ Supply chain's additional expenses for the effective implementation of joint forecasting activities, where $C_i^{CF}(t) = C^{CF}\{\alpha_5^i(t)\} = C_5^F\{\alpha_5^i(t)\} - C_1^F\{\alpha_1(t)\} - C_i^F\{\alpha_i(t)\}$.

$C_i^{CFM}(t)$ $C_i^{CFM}(t) = Max\{C_i^{CF}(t)\}$.

$\alpha_i(t)$ Forecasting capability (FC) for supply chain partners and the supply chain at time *t* (*i=1, 2, 3, 4,* & *5*).

$\Pi_i^{\alpha}(t)$ Expected profit of supply chain partner *i* (*i =1, 2, 3,* and *4*) before accounting for the forecasting collaboration expenses by time *t.*

$\Pi_5^{ia}(t)$ Expected supply chain profit before accounting for the joint forecasting collaboration costs by time t.

$\Pi_i(t)$ Expected profit of supply chain partner *i* by time *t*, where $\Pi_i(t) = \Pi_i^a(t) - C_i^F(t)$ for *i =1, 2, 3,* and *4*, and $\Pi_5(t) = \Pi_5^i(t) = \Pi_5^{ia}(t) - C_5^F(t)$.

$\Pi_5^{iaP}(t)$ Supply chain's profit portion gained by joint forecasting before accounting for the collaboration costs by time *t*, where $\Pi_5^{iaP} = \Pi_5^{ia} - (\Pi_1^a + \Pi_i^a)$.

$\Pi_5^{inP}(t)$ Supply chain's expected net profit gained by joint forecasting coordination by time *t*, where $\Pi^{inP} = \Pi_5^i - (\Pi_1 + \Pi_i) = \Pi_5^{iaP} - C_i^{CF}$.

3.1 Forecasting Capability (FC) in the Supply Chain

Demand forecasting is an essential component that prompts all the push and pull processes of the supply chain, but it is also one of the most challenging aspects of the supply chain process, primarily due to the volatile and uncertain nature of market demand. When forecasting errors and lack of collaboration overlap, an inherent mistrust develops between supply chain partners, creating conflicts throughout the supply chain processes. In order to profitably respond to and meet customer demand by matching demand with supply, firms in supply chains should develop their ability to create accurate and credible forecasts and efficiently manage the forecasting process using proven statistical forecasting methods. Forecasting capability (FC) increases as supply chain partners work together by investing in vital processes, systems, technology, skills, and training. Also, high FC levels can facilitate stronger ties and greater trust in buyer-supplier relationships (Chen et al., 2010; Fawcett et al., 2012).

3.2. Trust Level in Collaborative Demand Forecasting

Trust is generally considered a multi-layered concept constructed of various constituent parts such as the reliability of another party, the party's competency, the altruistic faith or goodwill felt toward another party, and vulnerability to trust (Riddalls et al., 2002; Ebrahim-Khanjari et al., 2013; Villena et al., 2019). In this paper, however, we view supply chain trust as a capability- and cost/benefit-based concept since companies operating in today's increasingly competitive global marketplace cannot afford to work with supply chain partners based on benevolence (Fawcett et al., 2012; Uca et al., 2017). Trust between supply chain partners is considered an iterative experience that evolves and develops or declines as a relationship advances through different phases (*Ö*zer et al., 2011; Brinkhoff et al., 2015). Collaborative demand forecast sharing, however, may require the partners to hold a sufficient degree of mutual trust not only to exchange a truthful demand forecast but also to counter fears of abuse of confidential forecast information and data. A high level of partner trust can create motivation for open communication and a willingness to share forecast information, technology, resources, and knowledge to achieve mutual goals (Sahay, 2003; Uca et al., 2017). Without a certain level of partner trust during the forecasting collaboration, the quality and precision of information exchanged between the buyer and the partner supplier can be reduced.

3.3. Supply Chain Forecasting Evolution Process

In this study, we consider a two-tier supply chain with one buyer (he) and one supplier (she) operating in an industry with relatively high demand volatility such as seasonal or short-term lifecycle products and use the single-period newsvendor framework to capture the supply-demand mismatch cost (Kurtulus et al., 2012). We also characterize the supplier based on her information accessibility and the degree of her dependence upon the buyer. Some suppliers may have sufficient and useful information about the buyer's market to create a demand forecast on their own initiative and develop their production plans to meet buyers' demand forecast. In contrast, others either have insufficient information or no information to project a demand forecast, so that they may have to make their production plans relying on buyers' forecast inputs. Hence, we consider three categories of suppliers with whom the buyer may establish forecasting coordination and use the subscript [*i*] to indicate the supply chain's trading partners. For notation simplicity in the modelling processes, we label the buyer as *1* (*i = 1*), the supplier who has all the necessary market information to make a demand forecast on her own as *2* (*i = 2*), the supplier who has only partial market demand information to create a demand forecast as *3* (*i = 3*), the supplier who has no information available for market demand as *4* (*i = 4*), and the supply chain where the buyer and his supplier collaborate to create a common demand forecast as *5* (*i = 5*). To reflect the demand forecast sharing relationship between the buyer and his supplier *i* in our model, we define the buyer's trust as his confidence in supplier *i's* reliability, competence, dependability, integrity, and responsibility. The buyer, however, may provide supplier *i* with somewhat distorted demand forecasts if he is not fully trusting in her. We also define supplier *i*'s trust as her confidence in the buyer's reliability, integrity, and accurate forecast information. A fully trusting supplier believes the buyer's demand forecast with certainty and willingly relies on his forecasting report to determine her production plan to meet his forecasted demand. The supplier may regard the buyer's forecasted demand as somewhat distorted information if she does not fully trust him. We assume that the buyer and his supplier *i* both possess the FC with their own experiences, knowledge and skills, and personal know-how. They make a subjective decision on partner trust level, respectively, and may share demand forecasting-related information based on their predetermined level of partner trust. We also view the level of trust in a buyer-supplier relationship as a continuous numerical value function to time t that can be adjusted at any time in response to the change in the other party's trust level. Unlike many studies (Cachon & Lariviere, 2001; Aviv, 2003; Özer & Wei, 2006; Ha & Tong, 2008; Li & Zhang, 2008; Shin & Tunca, 2010; Shang et al., 2016) that assume the level of trust as dichotomous – either absolute trust or no trust at all, we consider a continuum level of trust ranging from 0.0 to 1.0. The higher this value is, the higher the partners' willingness to share truthful demand information with their counter partners. For instance, we interpret the trust level of 0.0 as an absence of collaborative trust in the partner relationship. Specifically, the partners with no trust development exchange transactional, routine demand forecast only and do not share sensitive and private demand forecast information, maybe because of their exposure to potential vulnerability and the fear of the other party's opportunistic behavior. The trust level of 0.5 implies that the partner would be willing to share with the other party only 50% of the critical and credible forecasting information it may possess. The trust level of 1.0, on the other hand, refers to the complete willingness of the partner to exchange all the necessary and credible information and data and share with the other party its private and confidential demand forecasts without distorting them.

We now investigate the demand forecast evolution stochastic processes for the buyer, supplier *i*, and the coordinated supply chain. Let t_0 denote the initial time when sales start and let *T* be the terminal time when sales end (i.e., $\in [t_0, T]$). We denote the level of partner trust at time *t* by $r_i(t)$ and set its range as $0.0 \le r_i(t) \le 1.0$, where *i = 1, 2, 3,* and *4*. Let $\alpha_i(t) \in [0, \infty)$ denote the supply chain partner's FC at time *t*, for *i = 1, 2, 3,* and *4*. Let $\alpha_5(t) \in [0, \infty)$ denote the combined FC of the supply chain at time *t*, for *i = 5*. For the given forecasting volatility (σ_i), the combined FC is a function of r_1, α_1, r_i, and α_i and expressed as $\alpha_5(t) = \alpha_5^i(t) = f(r_1, \alpha_1, r_i, \alpha_i)$, where *i =2, 3,* and *4*. For example, if the buyer is coordinated with supplier *2*, the combined FC of that coordinated supply chain is expressed as $\alpha_5(t) = \alpha_5^2(t) = f(r_1, \alpha_1, r_2, \alpha_2)$. Let $\{d(t), t > 0\}$ denote the actual market demand at time *t* as a continuous numerical value function and $\left\{D(t) = \int_0^t d(x)dx\right\}$ denote the cumulative actual market demand by time *t*. For *i = 1, 2, 3, 4,* and *5*, let $F_i(t)$ be the cumulative forecasted market demand by time *t*, $K_i(t)$ be the expected value of $F_i(t)$, and $M_i(t)$ be the target value of a demand forecast. We assume that the partners in the supply chain will try to achieve $F_i(t)$ closest to $M_i(t)$. Trustful demand forecast sharing can help the supply chain partners create a demand forecast closest to the actual demand. However, the buyer and his supplier *i* may start their partnership with unequal trust levels toward their counter partner, and they may not necessarily hold the same levels of partner trust during the process of forecasting evolution. Moreover, the level of partner trust (r_i) may not play any role in some situations. For instance, in the buyer-supplier *2* relationship, the buyer trust level (r_1) is not critical in the supplier's decision because she is able to create a demand forecast on her own without the buyer's demand information. Since supplier *2* can collect market information and data similar to those acquired by the buyer, there would be no significant difference in the demand forecast target between them. Thus, we assume that the demand forecast target of supplier *2*, including the coordinated supply chain, would be the same as the buyer – that is, $M_1(t) = M_2(t) = M_5(t) = D(t)$. In the buyer-supplier *3* and the buyer-supplier *4* relationships, however, the buyer trust level (r_1) can influence the supplier's decision and, similarly, the supplier trust levels (r_3 and r_4) can influence the buyer's decision. One party in the supply chain would react to the other party's trust level and strategically adjust its willingness to collaborate accordingly. To reflect these issues into the determination of $M_i(t)$ for *i = 3* and *4*, we create two trust functions. We define $r_{1i}^T(t)$ as a function of $r_1(t)$ such that $r_{1i}^T(t) = f\{r_1(t)\} \in (0, \infty)$, which describes the buyer's trust level in his supplier *i* at time *t*, regarding how truthfully the buyer will be willing to share his forecasted demand information with his supplier – without distorting the demand forecast by inflating or contracting it. We define $r_{i1}^T(t)$ as a function of $r_i(t)$ and $r_{1i}^T(t)$ such that $r_{i1}^T(t) = f\{r_i(t), r_{1i}^T(t)\} \in (0, \infty)$, which describes the supplier *i*'s trust level in the buyer at time *t*, regarding how willingly the supplier will rely on the buyer's demand forecast and/or how she will adjust the demand forecast received from the buyer. We also consider the degree of supplier dependence on the buyer's demand forecast information, in addition to the effect of $\{r_{i1}^T(t)$, for $i = 3$ and $4\}$. Let w_1 be a weight for supplier *i*'s dependence on market demand information and forecasting data acquired by herself and w_2 be a weight for supplier *i*'s dependence on a demand forecast given by the buyer, where $w_1 + w_2 = 1$. Then, we have the following conditions: $0 < w_1, w_2 < 1$, for supplier *3*; $w_1 = 0$ and $w_2 = 1$, for supplier *4*. Based on the given set of $r_{i1}^T(t)$ and w_i, we now define supplier *3*' demand forecast target as $M_3(t) = E[w_1 D(t) + w_2 r_{31}^T(t) F_1(t)] = w_1 D(t) + w_2 r_{31}^T(t) K_1(t)$ and supplier *4*'s target as $M_4(t) = E[r_{41}^T(t) F_1(t)] = r_{41}^T(t) K_1(t)$.

In our forecasting evolution model, we utilize the Ornstein-Uhlenbeck process approach and the theoretical framework developed by Nam *et al.* (2011) to analyze how the supply chain partners can forecast a market demand under their FC at time *t*. Once the necessary adjustments to the demand forecast are made in an infinitesimal time interval during the forecasting process, the resulting tangible improvements are expected to be $\alpha_i(t)\{M_i(t) - F_i(t)\}dt$ for *i = 1, 2, 3, 4,* and *5*. A higher level of the FC speeds up corrections that lead to more accurate demand forecasting. To deal with the forecasting volatility, we apply the increment of a Wiener process to our models in the form of $\sigma_i dW(t)$. Let $F_i(t_0) = F_i^0$, $E[F_i(t_0)] = K_i(t_0) = K_i^0$, and $E[F_i^2(t_0)] = G_i = G_i(t_0) = G_i^0$. For *i = 1, 2,* and

5, let the initial condition of F_i^0 be either constant or Normally distributed with a mean of μ_i^0 and variance of σ_i^0. For i = 3 and 4, let $F_3^0 = w_1 r_{13}^T(t_0)F_1^0 + w_2 D(t_0)$, and $F_4^0 = r_{14}^T(t_0)F_1^0$. Our demand forecast evolution stochastic processes for i = 1, 2, 3, 4, and 5 can be defined as follows:

$$dF_i(t) = \alpha_i(t)\cdot\{M_i(t) - F_i(t)\}dt + \sigma_i dW(t) \quad (1)$$

For more details on the estimations of α_i and σ_i in Equation (1), see Arnold (1974), and Dixit and Pindyck (1994)

Lemma 1. If $F_i(t)$ follows Equation (1), where i = 1, 2, 3, 4, and 5, then the integral (in the Ito sense) of the infinitesimal diffusion and the variance of $F_i(t)$, can be obtained as follows:

$F_i(t) = e^{-R_i(t)}\left[F_i^0 + \int_0^t \alpha_i(x)M_i(x)e^{R_i(x)}dx + \sigma_i\int_0^t e^{R_i(x)}dW(x)\right]$,
$K_i(t) = E[F_i(t)]$ =$e^{-R_i(t)}\left\{K_i^0 + \int_0^t \alpha_i(x)M_i(x)e^{R_i(x)}dx\right\}$,
$G_i(t) = E\left[F_i^2(t)\right] = e^{-2R_i(t)}\left[G_i^0 + \int_0^t\{2\alpha_i(x)K_i(x)M_i(x) + \sigma_i^2\}e^{2R_i(x)}\,dx\right]$,
$Var_t(t) = E[F_i(t) - K_i(t)]^2 = G_i(t) - \{K_i(t)\}^2$, where $R_i(t) = \left\{\int_0^t \alpha_i(x)dx\right\}$.

Proof.

If $F_i(t)$ follows Equation (1), $F_i(t) = e^{-R_i(t)}\left[F_i^0 + \int_0^t \alpha_i(x)M_i(x)e^{R_i(x)}dx + \sigma_i\int_0^t e^{R_i(x)}dW(x)\right]$ Since $K_i(t)$ satisfies that $\dot{K}_i(t) = -\alpha_i(t)K_i(t) + \alpha_i M_i(t)$ with K_i^0, and $G_i(t)$ also satisfies that $\dot{G}_i(t) = -2\alpha_i(t)G_i(t) + 2K_i(t)\alpha_i(t)M_i(t) + \sigma_i^2$ with G_i^0, we have $K_i(t) = e^{-R_i(t)}\left\{K_i^0 + \int_0^t \alpha_1(x)M_1(x)e^{R_1(x)}dx\right\}$ and $G_i(t) = e^{-2R_i(t)}\left[G_i^0 + \int_0^t\{2\alpha_i(x)K_i(x)M_i(x) + \sigma_i^2\}e^{2R_i(x)}\,dx\right]$. Therefore, $Var_i(t) = E[F_i(t) - K_i(t)]^2 = G_i(t) - \{K_i(t)\}^2$. Based on the results of Lemma 1, we determine the values of the expected demand for each supply chain partner by time t and the forecasting variance associated with demand volatility. Let $I_B(w) = 1$ if $w \in B$ and $I_B(w) = 0$ if $w \notin B$. For the buyer (i = 1) and the coordinated supply chain (i = 5), let $B_{i1}(t) = \{t|F_i(t) \le D(t)\}$ and $B_{i2}(t) = \{t|F_i(t) > D(t)\}$. For supplier 2, 3, and 4, let $B_{i1}(t) = \{t|F_i(t) \le F_1(t)\}$, and $B_{i2}(t) = \{t|F_i(t) > F_1(t)\}$. Let $N_i(t)$ be the probability that the demand forecast by the buyer and the coordinated supply chain is less than or equal to actual market demand and that the demand forecast by supplier i is less than or equal to the buyer's demand forecast and denote it by:

$$N_i(t) = P\{B_{i1}(t)\} = [1/\left(2\pi H_i(t)\right)^{1/2}]\left[\int_{-\infty}^{Z_i(t)} exp\{-x^2/2H_i(t)\}\,dx\right] \text{ for } i = 1, 2, 3, 4 \text{ and } 5$$

Lemma 2. For a given set of $\alpha_i(t)$ and σ_i,
a) For i = 1 and 5, $\left[I_{B_{i1}}(t)\right] = P\{B_{i1}(t)\} = N_i(t)$, $E\left[I_{B_{i2}}(t)\right] = P\{B_{i2}(t)\} = 1 - N_i(t)$, where $H_i(t) = \int_0^t exp\{2R_i(x)\}\,dx$, and $Z_i(t) = (1/\sigma_i)[D(t)exp\{R_i(t)\} - F_i^0 - \int_0^t \alpha_i(x)D(t)\exp\{R_i(x)\}\,dx]$.
b) For i=2, 3, and 4,$\left[I_{B_{i1}}(t)\right] = P\{B_{i1}(t)\} = N_i(t)$, $E\left[I_{B_{i2}}(t)\right] = P\{B_{i2}(t)\} = 1 - N_i(t)$, $H_i(t) = \sigma_1^2 exp\{-2R_1(t)\}\int_0^t exp\{2R_1(x)\}\,dx + \sigma_i^2 exp\{-2R_i(t)\}\int_0^t exp\{2R_i(x)\}\,dx$ and
$Z_i(t) = exp\{-R_1(t)\}\left\{F_1^0 + \int_0^t \alpha_1(x)D(x)exp\{R_1(x)\}\,dx\right\}$ $+exp\{-R_1(t)\}\left\{F_1^0 + \int_0^t \alpha_i(x)M_i(x)exp\{R_i(x)\}\,dx\right\}$.

Proof.
a) Since $P[F_i(t) \le D(t)] = P\left[\int_0^t e^{R_i(x)}dW(x) \le \left(\frac{1}{\sigma_i}\right)\left\{D(t)e^{R_i(t)} - F_i^0 - \int_0^t \alpha_i(x)D(x)e^{R_i(x)}dx\right\}\right]$ = $\left\{\frac{1}{\sqrt{2\pi H_i(t)}}\right\}\left[\int_{-\infty}^{Z_i(t)} e^{\frac{-x^2}{2H_i(t)}}\,dx\right]$, we have $P\{B_{i1}(t)\} = N_i(t)$ and $P\{B_{12}\} = P\{F_i(t) > D(t)\}$
$= 1 - P\{B_{i1}\} = 1 - N_i(t)$, where
$H_i(t) = \int_0^t e^{2R_i(x)}dx$ and $Z_i(t) = \left(\frac{1}{\sigma_i}\right)\left\{D(t)e^{R_i(t)} - F_i^0 - \int_0^t \alpha_i(x)D(x)e^{R_i(x)}dx\right\}$
b) Since $P[F_i(t) \le F_1(t)]$, we have $P\left[\sigma_i e^{-R_i(t)}\int_0^t e^{R_i(x)}dW(x) - \sigma_1 e^{-R_1(t)}\int_0^t e^{R_1(x)}dW(x) \le e^{-R_1(t)}\left\{F_1^0 + \int_0^t \alpha_1(x)D(x)e^{R_1(x)}dx\right\} - e^{-R_i(t)}\left\{F_i^0 + \int_0^t \alpha_i(x)M_i(x)e^{R_i(x)}dx\right\}\right]$ $= \frac{1}{\sqrt{2\pi H_i(t)}}\int_{-\infty}^{Z_i(t)} e^{\frac{-x^2}{2H_i(t)}}dx$, where $H_i(t) = \sigma_1^2 e^{-2R_1(t)}\int_0^t e^{2R_1(x)}dx + \sigma_i^2 e^{-2R_i(t)}\int_0^t e^{2R_i(x)}dx$. $Z_i(t) = e^{-R_1(t)}\left\{F_1^0 + \int_0^t \alpha_1(x)D(x)e^{R_1(x)}dx\right\} - e^{-R_i(t)}\left\{F_i^0 + \int_0^t \alpha_i(x)M_i(x)e^{R_i(x)}dx\right\}$.

Hence, $P(B_{i1}) = P[F_i(t) \leq F_1(t)] = N_i(t)$, and $P\{B_{i2}\} = P\{F_1(t) > F_i(t)\} = 1 - P\{B_{i1}\} = 1 - N_i(t)$.
Through Lemma 2, we can calculate the probabilities of overstock and understock for the buyer, supplier i, and the coordinated supply chain. Let $\emptyset(x) = \left(\frac{1}{\sqrt{2\pi}}\right) e^{-x^2/2}$ and $\Phi(t) = \int_{-\infty}^{t} \phi(x)dx$. For i = *1* and *5*, $A_{i1}(t) = K_i(t) - \sqrt{Var_i(t)}\left[\frac{\emptyset\{u_i(t)\}}{\Phi\{u_i(t)\}}\right]$, $A_{i2}(t) = K_i(t) + \sqrt{Var_i(t)}\left[\frac{\emptyset\{u_i(t)\}}{1-\Phi\{u_i(t)\}}\right]$, and $u_i(t) = \frac{D(t)-K_i(t)}{\sqrt{Var_i(t)}}$. For i = *2, 3* and *4*, $A_{i1}(t) = \{K_i(t) - K_1(t)\} - \sqrt{Var_1(t) + Var_i(t)}\left[\frac{\emptyset\{u_i(t)\}}{\Phi\{u_i(t)\}}\right]$, $A_{i2}(t) = \{K_i(t) - K_1(t)\} + \sqrt{Var_1(t) + Var_i(t)}\left[\frac{\emptyset\{u_i(t)\}}{1-\Phi\{u_i(t)\}}\right]$, and $u_i(t) = \frac{\{K_1(t)-K_i(t)\}}{\sqrt{Var_1(t)+Var_i(t)}}$. From now on, when no confusion arises, the time index t will be suppressed for the sake of simple exposition.

Proposition 1. For a given set of $\alpha_i(t)$ and σ_i, where $0 \leq t \leq T$,
a) For i = *1* and *5*, $E[F_i|B_{i1}] = A_{i1}$ and $E[F_i|B_{i2}] = A_{i2}$.
b) For i = *2, 3,* and *4*, $E[F_i - F_1|B_{i1}] = A_{i1}$ and $E[F_i - F_1|B_{i2}] = A_{i2}$.

Proof.

a) Since F_i is a Gaussian stochastic process with mean (K_i) and variance (Var_i), we have $E[F_i|B_{i1}] = E[F_i|F_i \leq D] = A_{i1} = K_i - \sqrt{Var_i}\left[\frac{\emptyset\{u_i\}}{\Phi\{u_i\}}\right]$, $A_{i2} = K_i + \sqrt{Var_i}\left[\frac{\emptyset\{u_i\}}{1-\Phi\{u_i\}}\right]$ by the truncated Normal distribution theory (Arnold, 1974 and Ryan, 2000). In the same way, $A_{i2} = E[F_i|B_{i2}] = E[F_i|F_i > D] = K_i + \sqrt{Var_i}\left\{\frac{\emptyset\{u_i\}}{1-\Phi\{u_i\}}\right\}$, where $u_i = \frac{D-K_i}{\sqrt{Var_i}}$, for i=*1* and *5*.
b) Let $X = F_i - F_1$, then $E[F_i - F_1|F_i - F_1 < 0] = E[X|X < 0]$. X is a Gaussian stochastic process with mean $(K_i - K_1)$ and variance $(Var_1 + Var_i)$. Hence, $A_{i1} = E[F_i - F_1|F_i - F_1 < 0] = E[X|X < 0] = (K_i - K_1) - \sqrt{Var_1 + Var_i}\left[\frac{\emptyset\{u_i\}}{\Phi\{u_i\}}\right]$, and $A_{i2} = E[F_i - F_1|F_i - F_1 > 0] = E[X|X > 0] = (K_i - K_1) + \sqrt{Var_1 + Var_i}\left[\frac{\emptyset\{u_i\}}{1-\Phi\{u_i\}}\right]$, where $u_i = \frac{(K_1-K_i)}{\sqrt{Var_1+Var_i}}$, for i = *2, 3,* and *4.*

Proposition 2. For a given set of $\alpha_i(t)$ and σ_i, where $0 \leq t \leq T$,
a) For i = *1* and *5*, $E[F_i I_{B_{i1}}] = A_{i1}N_i$ and $E[F_i I_{B_{i2}}]$ =$\{1 - N_i(t)\}A_{i2}$.
b) For i = *2, 3* and *4*, $E[(F_i - F_1)I_{B_{i1}}(t)] = A_{i1}N_i$(t) and $E[(F_i - F_1)I_{B_{i2}}]$ =$\{1 - N_i(t)\}A_{i2}$.
Proof.
a) For i =*1* and *5*, since $E[F_i I_{B_{i1}}] = E[F_i|B_{i1}]P[B_{i1}]$ (Ash, 1972 and Chung 1974), we have $E[F_i I_{B_{i1}}] = A_{i1}N_i$ by Lemma 1 and Lemma 2.
b) By the same way, we have $[(F_i - F_1)I_{B_{i1}}] = E[F_i - F_1|F_i - F_1 < 0]P[B_{i1}] = A_{i1}N_i$. And $E[(F_i - F_1)I_{B_{i2}}] = E[(F_i - F_1)|B_{i2}]P[B_{i2}]$ =$\{1 - N_i\}A_{i2}$, for i =*2, 3,* and *4.*

In Proposition 1, part (a) describes the expected market demands for i =*1* and *5* under the given conditions that the buyer's and the coordinated supply chain's demand forecasts are less than or equal to the actual demand. Part (b) illustrates the expected overstock or understock of supplier i when the buyer's demand forecast is higher than or equal to the one projected by the coordinated supply chain. Proposition 2 provides the equations needed to quantify the expected profits for the buyer, supplier i, and the coordinated supply chain.

3.3.1. *Expected Loss due to Overstock and Understock:*

In this section, we consider the expected costs for overstock and understock resulting from forecasting errors. Let P be the retail price per unit of the buyer, C_V be the unit production cost of supplier i, C be the wholesale price of supplier i for the buyer, C_i^U be the cost for understocking, C_i^O be the cost for overstocking, and S_b and S_S be the salvage values for the buyer and supplier i at the end of the sales season, respectively. The deviation of the production quantity from the actual demand causes economic losses associated with understock and overstock. $C_i^U\{D - F_i\}I_{B_{i1}}$ and $C_i^O\{F_i - D\}I_{B_{i2}}$ are the penalty costs for understock and overstock imposed on the buyer and the coordinated supply chain, respectively. $C_i^O\{F_i - F_1\}I_{B_{i2}}$ and $C_i^U\{F_1 - F_i\}I_{B_{i1}}$ are the penalty costs for overstock and understock imposed on supplier i, respectively. Let $l_i(t)$ be the total penalty cost due to overstock and understock, and $L_i(t) = E\{l_i(t)\}$, for i = *1, 2, 3, 4,* and *5*.

Proposition 3. For a given set of $\alpha_i(t)$ and σ_i, where $0 \leq t \leq T$,

a) For i =*1* and *5*, $L_i(t) = E\{l_i(t)\} = N_i \cdot \{D \cdot (C_i^O + C_i^U) - C_i^U A_{i1} - C_i^O A_{i2}\} + C_i^O(A_{i2} - D)$.

b) For *i = 2, 3,* and *4*, $L_i(t) = E\{l_i(t)\} = C_i^O A_{i2} - N_i \cdot \{C_i^U A_{i1} + C_i^O A_{i2}\}$.

Proof.

a) Since $D(t)$ is assumed to be a bounded continuous numerical value function at time *t*, we have $E\{DI_{B_{i1}}\} = DE\{I_{B_{i1}}\}$. Because $E[\{D - F_i\}I_{B_{i1}}] = E\{DI_{B_{i1}}\} - E\{F_i I_{B_{i1}}\} = D \cdot E\{I_{B_{i1}}\} - A_{i1}N_i = DN_i - A_{i1}N_i = N_i(D - A_{i1})$ and $E\{(F_i - D)I_{B_{i2}}\} = E\{F_i I_{B_{i2}}\} - E\{DI_{B_{i2}}\} = (1 - N_i)A_{i2} - (1 - N_i)D = (1 - N_i)(A_{i2} - D)$, we have $L_i(t) = N_i \cdot \{D \cdot (C_i^O + C_i^U) - C_i^U A_{i1} - C_i^O A_{i2}\} + (A_{i2} - D)C_i^O$ by Proposition 1 and Proposition 2, for *i =1* and *5*.

b) Let $X = F_i - F_1$. $L_i(t) = E[l_i(t)] = E[(-C_i^U)(X)I_{\{X \le 0\}} + (C_i^O)(X)I_{\{X>0\}}] = (-C_i^U)E[XI_{\{X \le 0\}}] + (C_i^O)E[XI_{\{X>0\}}] = C_i^O A_{i2} - C_i^O A_{i2}N_i - C_i^U A_{i1}N_i = C_i^O A_{i2} - \{C_i^O A_{i2}(t) + C_i^U A_{i1}\}N_i$, for *i=2, 3,* and *4*.

Proposition 3 shows the expected profit losses resulting from over- and under-demand forecast estimations for the buyer, supplier *i*, and the coordinated supply chain.

3.4. Profitability Analysis

This section first attempts to estimate individual profits for the buyer and supplier *i* under no involvement in collaborative demand forecast sharing between them and then develop the expected profit functions for the coordinated supply chain based on the combined FC and partner trust levels. Let $C^F\{\alpha_i(t)\}$ be the forecasting expenses for each partner and the coordinated supply chain (*i = 1, 2, 3, 4,* and *5*). We substitute C_i^F for $C^F\{\alpha_i(t)\}$ for notation simplicity (i.e., $C^F\{\alpha_i(t)\} = C_i^F$). Let Π_i be the expected profit after accounting for the forecasting expenses for *i = 1, 2, 3, 4,* and *5* by time *t*. Let $\Pi_i^a(t)$ be the expected profit of the supply chain partner *i* (*i = 1, 2, 3,* and *4*) before accounting for the forecasting expenses and $\Pi_5^{ia}(t)$ be the coordinated supply chain's expected profit before accounting for the joint forecasting collaboration costs, respectively. To analyze the profit functions for the supply partners and the coordinated supply chain in the following sections, we set that $J_i(t) = A_{i1}(t)(P + C_i^U) + A_{i2}(t)C_i^O - (C_i^O + C_i^U + P)D(t)$, where *i = 1* and *5*, and that $J_i(t) = C_i^O A_{i2}(t) + (C + C_i^U)A_{i1}(t)$, where *i = 2, 3,* and *4*. We denote the buyer's profit as $\Pi_1^a(t) = (t)P + N_1(t)J_1(t) - C_1^O \cdot \{A_{12}(t) - D(t)\} - CK_1(t)$. For the supplier *i*'s profit, $\Pi_i^a(t) = CK_1(t) + N_i(t)J_i(t) - C_V K_i(t) - C_i^O A_{i2}(t)$, for *i = 2, 3,* and *4*. For the coordinated supply chain's profit, $\Pi_5^{ia}(t) = D(t)P + N_5(t)J_5(t) - C_5^O \cdot \{A_{52}(t) - D(t)\} - C_V K_5(t)$.

3.4.1 The Buyer's Expected Profit:

The buyer operating under the supply chain with no forecasting coordination creates a demand forecast based on his own FC and provides his ordering decisions to his supplier, whose production capacity is enough to meet the buyer's order quantity. Let $BP(t)$ be the buyer's profit by time *t*. For a given set of α_1 and σ_1, the buyer's expected profit is written as $\Pi_1(t) = E[BP(t)]$, where $M_1(t) = D(t)$ and $K_1(t) = e^{-R_2(t)}\left\{K_2^0 + \int_0^t \alpha_1(x)D(x)e^{R_1(x)}dx\right\}$.

Proposition 4. For a given set of α_1 and σ_1, the buyer's expected profit by time *t* is expressed as follows:

$$\Pi_1 = \Pi_1^a - C_1^F. \qquad (2)$$

Proof.

The buyer's revenue is calculated as either PF_1 if $D \ge F_1$ or $D_1 P$ if $D < F_1$. Then, the buyer's profit (BP) is written as $DP - (D - F_1)PI_{[F_1 < D]} - l - CF_1 - C_1^F$. The buyer's expected profit by time *t* for a given set of α_1 and σ_1 can be expressed as $E[BP] = DP - E\{P(D - F_1)I_{B_1}\} - E[l] - CE[F_1] - C_1^F = DP - DPN_1 + PA_{11}N_1 - \{(C_1^O + C_1^U)D - C_1^U A_{11} - C_1^O A_{12}\}N_1 - (A_{12} - D)C_1^O - CK_1 - C_1^F$ by Proposition 3. Thus, $\Pi_1 = DP + N_1 J_1 - (A_{12} - D)C_1^O - CK_1 - C_1^F = \Pi_1^a - C_1^F$.

3.4.2. *The Supplier's Expected Profit under Three Scenarios:*

Considering the role of partner trust level in demand forecasting collaboration and the degree of supplier *i*'s dependence on the buyer's forecast inputs, we attempt to develop a framework to estimate the supplier *i*'s expected profit (*i = 2, 3,* and *4*). Let $SP_i(t)$ be supplier *i*'s profit by time *t* and $\theta_i(t)$ be a set of the supply chain partners' FC and forecasting volatility such that $\theta_i(t) = \{\alpha_1(t), \sigma_1(t), \alpha_i(t), \sigma_i(t)\}$, for *i = 2, 3,* and *4*. For a given set of θ_i and $r_{i1}^T(t)$, the expected profit of supplier *i* is written as $\Pi_i(t) = E[SP_i(t)]$.

Proposition 5. For a given set of θ_i and r_{i1}^T, supplier *i*'s expected profit by time *t* can be expressed as follows:

$$\Pi_i = \Pi_i^a - C_i^F, \text{ where } i = 2, 3, \text{ and } 4.$$

Proof.

Let $F_1 - F_i = X_i$. Supplier *i*'s revenue is defined as either CF_i, if $F_1 \geq F_i$ or CF_1, if $F_1 < F_i$. Supplier *i*'s profit is written as $SP_i = CF_1 - C\{F_1 - F_i\}I_{[F_i<F_1]} - l_i - C_V F_i - C_i^F$. Therefore, supplier *i*'s expected profit is expressed as $E(SP_i) = CK_1 - CE\{XI_{[X_i<0]}\} - E[l_i] - C_V E[F_i] - C_i^F = CK_1 + N_i J_i - C_V K_i - C_i^O A_{i2} - C_i^F$ by Proposition 3. Hence, $\Pi_i = CK_1 + N_i J_i - C_V K_i - C_i^O A_{i2} - C_i^F = \Pi_i^a - C_i^F$.

To summarize, for a given set of θ_i and r_{i1}^T, we can estimate the expected profit by time *t* for the supplier of three types (*i* = *2, 3,* and *4*) by utilizing the following Equation,

$$\Pi_i = CK_1 + N_i J_i - C_V K_i - C_i^O A_{i2} - C_i^F = \Pi_i^a - C_i^F, \quad (3)$$

where $M_2(t) = D(t)$, $M_3(t) = w_1 r_{31}^T(t) K_1(t)$, $M_4(t) = r_{41}^T(t) K_1(t)$, $K_2(t) = e^{-R_2(t)}\left\{K_2^0 + \int_0^t \alpha_2(x) D(x) e^{R_2(x)} dx\right\}$, $K_3(t) = e^{-R_3(t)}\left\{K_3^0 + \int_0^t \alpha_3(x) w_1 r_{31}^T(t) K_1(x) e^{R_3(x)} dx + \int_0^t \alpha_3(x) w_2 D(x) e^{R_3(x)} dx\right\}$ with $r_{31}^T(t) = f\{r_3(t), r_{13}^T(t)\}$, $0 < w_1,\ w_2 < 1$, and $w_1 + w_2 = 1$, $K_4(t) = e^{-R_4(t)}\left\{K_4^0 + \int_0^t \alpha_4(x) r_{41}^T(t) K_1(x) e^{R_4(x)} dx\right\}$ with $r_{41}^T(t) = f\{r_4(t), r_{14}^T(t)\}$.

3.4.3. *The Coordinated Supply Chain's Expected Profit:*

In the supply chain (*i* = *5*), the buyer and supplier *i* work together to share common demand forecast information and continuously update and revise their demand forecasts to reach a final demand forecast close to actual demand. Considering that the buyer and supplier *i* may have different FC and trust levels, we develop the coordinated supply chain's expected profit function. Let *SCP*(*t*) denote the coordinated supply chain's profit by time *t* under collaborative forecasting coordination with a given set of r_i and α_i. The coordinated supply chain's expected profit is written as $\Pi_5^i(t) = E[SCP(t)]$. However, successful implementation of joint forecasting activities may require both parties (or one party) to make additional investments in some areas for mutual benefits such as new forecasting information technology, upgraded support systems, and employee training for new skills. Therefore, we also consider the added cost for additional investments in forecasting collaboration and examine how this additional expense can affect the supply chain's profitability. Let $C^{CF}\{\alpha_5^i(t)\}$ be the supply chain's additional expenses due to the implementation of joint forecasting activities. The supply chain's additional expenses are written as $C^{CF}\{\alpha_5^i(t)\} = C_5^F\{\alpha_5^i(t)\} - C_1^F\{\alpha_1(t)\} - C_i^F\{\alpha_i(t)\}$, for *i* = *2, 3,* and *4*. We henceforth replace $C^{CF}\{\alpha_5^i(t)\}$ with $C_i^{CF}(t)$. Let C_i^{CFM} denote the maximum additional forecasting cost limit for the coordinated supply chain to achieve higher profit than the supply chain with no forecasting coordination and let $C_i^{CFM} = Max(C_i^{CF})$.

Proposition 6. For a given set of θ_i and α_5^i, the supply chain's expected profit (Π_5^i) when the buyer and his supplier *i* (*i* = *2, 3,* or *4*) collaborate in demand forecasting activities is expressed as follows:

$$\Pi_5^i = DP + N_5 J_5 - (A_{52} - D)C_5^O - C_V K_5 - C_5^F = \Pi_5^{ia} - C_5^F, \quad (4)$$

where $M_5(t) = D(t)$ and $K_5(t) = e^{-R_5(t)}\left\{K_5^0 + \int_0^t \alpha_5^i(x) D(x) e^{R_5(x)} dx\right\}$.

Proof.

Since SCP can be expressed as $P - P(D - F_5)I_{B_{51}} - l_5 - C_V F_5 - C_5^F$, we have $E[SCP] = \Pi_5^i = DP - PDN_5 + PA_{51}N_5 - \{D(C_5^O + C_5^U) - C_5^U A_{51} - C_5^O A_{52}\}N_5 - (A_{52} - D)C_5^O - C_V K_5 - C_5^F = DP + N_5 J_5 - (A_{52} - D)C_5^O - C_V K_5 - C_5^F$ by Proposition 3.

We now investigate the relationship between supply chain profit before accounting for the cost due to forecasting collaboration (Π_5^{ia}) and the combined FC. We also examine that under what conditions of (r_1, r_i), the coordinated supply chain's profitability can be maximized.

Theorem 1. The coordinated supply chain's expected profit before accounting joint forecasting cost (Π_5^{ia}) increases as its combined FC improves.

Proof. By applying the Ornstein-Uhlenbeck stochastic processes to our forecasting evolution model, we havelim$_{\alpha_5^i\to\infty}\{D(t) - F_5(t)\} = 0$, $\lim_{\alpha_5^i\to\infty} l_5(t) = 0$. Hence, the supply chain's penalty cost, $l_5(t)$, is a decreasing functio of $\alpha_5^i(t)$. If $D > F_5$, $(D - F_5)$ is a decreasing function of α_5^i. Because $(C_V - PI_{B_{51}}) < 0$, $(D - F_5)(C_V - PI_{B_{51}})$ is an increasing function of $\alpha_5^i(t)$. If $D < F_5$, then $(D - F_5)$ is an increasing function of $\alpha_5^i(t)$. Because

$(C_V - PI_{B_{51}}) > 0$, $(D - F_5)(C_V - PI_{B_{51}})$ is an increasing function of $\alpha_5^i(t)$. Since $D(P - C_V) + (D - F_5)(C_V - PI_{B_{51}}) - l_5(t)$ is an

increasing function of α_5^i, the expected value of $[D(P - C_V) + (D - F_5)(C_V - PI_{B_{51}}) - l_5(t)]$ is an increasing function of α_5^i, where *i = 2, 3,* and *4.*

Theorem 1 implies that the coordinated supply chain's profit (Π_5^{ia}) can grow as long as α_5^i is an increasing function of the integrated partner trust level.

Corollary 1. For a given θ_i, if the combined FC is an increasing function of partner trust levels, and if either $(\partial C_5^F/\partial\alpha_5^i) \leq 0$ or $(\partial\Pi_5^{ia}/\partial\alpha_5^i) > (\partial C_5^F/\partial\alpha_5^i)$, then the expected profit (Π_5^i) increases as the trust levels increase, and it is maximized when $r_1(t) = 1.0$ and $r_i(t) = 1.0$, for *i = 2, 3,* and *4.*

Proof.

Since $\Pi_5^i = \Pi_5^{ia} - C_5^F$, $(\partial\Pi_5^i/\partial r_i) = \{(\partial\Pi_5^{ia}/\partial\alpha_5^i) - (\partial C_5^F/\partial\alpha_5^i)\}(\partial\alpha_5^i/\partial r_i)$. We know that $(\partial\alpha_5^i/\partial r_i) > 0$ by the assumption and $\partial\Pi_5^{ia}/\partial\alpha_5^i > 0$ by Theorem 1. Therefore, if $(\partial C_5^F/\partial\alpha_5^i) \leq 0$ or $(\partial\Pi_5^{ia}/\partial\alpha_5^i) - (\partial C_5^F/\partial\alpha_5^i) > 0$, then $\partial\Pi_5^i/\partial r_i > 0$ for *i= 2, 3* and *4.* Hence, the coordinated supply chain profit (Π_5^i) has a maximum value at $r_1(t) = 1.0$ and $r_i(t) = 1.0$, for *i = 2, 3,* and *4.*

Corollary 1 shows the conditions to maximize supply chain profitability. If the forecasting collaboration cost does not increase by the improvement of the combined FC and also the marginal additional profit rate $(\partial\Pi_5^{iaP}/\partial\alpha_5^i)$ increases faster than the marginal additional cost rate $(\partial C_i^{CF}/\partial\alpha_5^i)$ as the combined FC improves, the coordinated supply chain profit is maximized when $r_1(t) = 1.0$ and $r_i(t) = 1.0$, for *i = 2, 3,* and *4.*

Considering the additional cost of joint forecasting collaboration (C_i^{CF}), we now explore how the trust-based forecasting collaboration can impact supply chain surplus and identify the optimal level of partner trust that enables the profit maximization of the coordinated supply chain. Let Π_5^{iaP} denote the supply chain's expected additional profit portion gained by the joint forecasting coordination before accounting for the collaboration costs, which is the profit difference between the supply chain with forecasting coordination and the one with no coordination such that $\Pi_5^{iaP} = \Pi_5^{ia} - (\Pi_1^a + \Pi_i^a)$. Let Π_5^{inP} denote the supply chain's expected additional net profit due to the joint demand forecasting coordination after accounting for the collaboration costs such that $\Pi_5^{inP} = \Pi_5^i - (\Pi_1 + \Pi_i) = \Pi_5^{iaP} - C_i^{CF}$. Let $\lambda_i = (\partial\Pi_5^{iaP}/\partial\alpha_5^i) - (\partial C_i^{CF}/\partial\alpha_5^i)$, $\lambda_{ii} = (\partial\lambda_i/\partial\,\alpha_5^i) = (\partial^2\Pi_5^{iaP}/\partial{\alpha_5^i}^2) - (\partial^2 C_i^{CF}/\partial{\alpha_5^i}^2)$, $V_1 = (\partial\alpha_5^i/\partial r_1)$, $V_{11} = (\partial V_1/\partial\, r_1) = (\partial^2\alpha_5^i/\partial{r_1}^2)$, $V_i = (\partial\alpha_5^i/\partial r_i)$, $V_{ii} = (\partial V_i/\partial\, r_i) = (\partial^2\alpha_5^i/\partial{r_i}^2)$, $V_{1i} = (\partial V_1/\partial\, r_i) = (\partial^2\alpha_5^i/\partial r_1\,\partial r_i)$. Let $r_{1i}^* = \{r_1^* \in [0.0,1.0]$ and $r_i^* \in [0.0,1.0]\}$. Let r_{1i}^* satisfy both $(\partial\alpha_5^i/\partial r_1)|_{r_{1i}^*} = V_1|_{r_{1i}^*} = 0$ and $(\partial\alpha_5^i/\partial r_i)|_{r_{1i}^*} = V_i|_{r_{1i}^*} = 0$, where *i = 2, 3,* and *4.*

Corollary 2. For a given θ_i, if the combined FC is an increasing function of partner trust levels and $\Pi_5^{iaP} > C_i^{CF}$, and if either $\lambda_i > 0$ or $(\partial C_i^{CF}/\partial\alpha_5^i) \leq 0$, for *i = 2, 3,* and *4*, the coordinated supply chain profit (Π_5^i) is always higher than the supply chain with no demand forecast sharing, and its expected additional net profit (Π_5^{inP}) is maximized at $r_1(t) = r_i(t) = 1.0$, for *i = 2, 3,* and *4.*

Proof.

For a given θ_i, if $\Pi_5^{iaP}(t) - C_i^{CF}(t) > 0$, then, $\Pi_5^{inP}(t) = \Pi_5^i - (\Pi_1 + \Pi_i) > 0$. Hence, $\Pi_5^i > (\Pi_1 + \Pi_i)$. Since $\Pi_5^{inP} = \Pi_5^{iaP} - C_i^{CF}$, we have $(\partial\Pi_5^{inP}/\partial r_i) = (\lambda_i)\,(\partial\alpha_5^i/\partial r_i)$. If either $(\lambda_i) > 0$ or $(\partial C_i^{CF}/\partial\alpha_5^i) \leq 0$, for *i = 2, 3,* and *4*, then $(\partial\Pi_5^{inP}/\partial r_i) > 0$. Thus, $\Pi_5^{inP}(t)$ is an increasing function of trust level and has a maximum value when $r_1(t) = 1.0$ and $r_i(t) = 1.0$, for *i = 2, 3,* and *4.*

Corollary 2 provides the conditions under which the supply chain with collaborative demand forecast sharing can generate higher profitability than when the buyer and supplier *i* in the supply chain maintain separate demand forecasts. It also shows that the coordinated supply chain's expected additional net profit is maximized when both parties hold the absolute level of partner trust throughout their working relationship.

Theorem 2. For a given θ_i, if α_5^i is a concave function of trust levels (r_1, r_i) and $\lambda_i > 0$, then the expected additional net profit of the supply chain (Π_5^{inP}) is maximized at $r_{1i}^* = (r_1^*,\ r_i^*)$.

Proof.

If α_5^i is a concave function of trust levels (r_1, r_i), then the combined FC is maximized at $r_{1i}^* = (r_1^*, r_i^*)$, $V_1|_{r_{1i}^*} = 0, V_i|_{r_{1i}^*} = 0$, $V_{11}|_{r_{1i}^*=(r_1^*,r_i^*)} < 0$, $V_{ii}|_{r_{1i}^*=(r_1^*,r_i^*)} < 0$, and $\{V_{11}V_{ii} - (V_{1i})^2\}|_{r_{1i}^*=(r_1^*,r_i^*)} > 0$. Since $\Pi_5^{inP} = \Pi_5^{iaP} - C_i^{CF}$, we have $\partial\Pi_5^{inP}/\partial r_1 = \lambda_i V_1, \partial\Pi_5^{inP}/\partial r_i = \lambda_i V_i$, $\partial^2\Pi_5^{inP}/\partial r_1^2 = (\lambda_{ii})(V_1)^2 + (\lambda_i)(V_{11})$, $\partial^2\Pi_5^{inP}/\partial {r_i}^2 = (\lambda_{ii})(V_i)^2 + (\lambda_i)(V_{ii})$, and $\partial^2\Pi_5^{inP}/\partial r_1 \partial r_i = \lambda_{ii}V_1V_i + (\lambda_i)(V_{1i})$. Hence, $\left(\partial\Pi_5^{inP}/\partial r_1\right)\big|_{r_{1i}^*} = \lambda_i V_1|_{r_{1i}^*} = 0, \partial\Pi_5^{inP}/\partial r_i = \lambda_i V_i|_{r_{1i}^*} = 0$. If $\lambda_i > 0$, then $\left(\partial^2\Pi_5^{inP}/\partial {r_1}^2\right)\big|_{r_{1i}^*} = (\lambda_i)(V_{11})|_{r_{1i}^*} < 0$, and $\left(\partial^2\Pi_5^{inP}/\partial {r_i}^2\right)\big|_{r_{1i}^*} = (\lambda_i)(V_{ii})|_{r_{1i}^*} < 0$. Hessian determinant of Π_5^{inP} at r_{1i} can be expressed as follows: $\mathcal{H} = \begin{vmatrix} (\lambda_i)(V_{11})|_{r_{1i}^*} & (\lambda_i)(V_{1i})|_{r_{1i}^*} \\ (\lambda_i)(V_{i1})|_{r_{1i}^*} & (\lambda_i)(V_{ii})|_{r_{1i}^*} \end{vmatrix} = (\lambda_i)^2\{V_{11}V_{ii} - (V_{1i})^2\}|_{r_{1i}^*=(r_1^*,r_i^*)} > 0$, because $\{V_{11}V_{ii} - (V_{1i})^2\}|_{r_{1i}^*=(r_1^*,r_i^*)} > 0$. Hence, Π_5^{inP} has a maximum at $r_{1i}^* = (r_1^*, r_i^*)$.

Theorem 2 provides some necessary conditions to determine an optimal level of partner trust that enables profit maximization of the supply chain. For the supply chain where its FC is concave on trust levels, and the marginal additional profit rate is higher than the marginal additional cost rate, the coordinated supply chain can generate higher profit than the supply chain with no coordination and achieve its expected maximum additional net profit at $r_{1i}^* = (r_1^*,\ r_i^*)$.

4. RESULTS with NUMERICAL ANALYSIS

This section conducts a numerical analysis to illustrate the impact of trust level-based FC on supply chain profitability. To compare the performance outputs of the supply chain with and without demand forecast sharing, we first investigate the expected profits for the buyer and his supplier *i* (*i* = *2*, *3*, and *4*) under no collaborative forecasting relationship between them and then evaluate the profitability of the coordinated supply chain with demand forecast sharing (*i* = *5*). Based on the Equations developed in the model section, we estimate maximum forecasting coordination costs and expected profits from forecasting collaboration. Let the sales start at the time $t_0 = 0$ and end at time *T* = 4. For the actual demand at time *t*, let $d(t) = 100t^3 - 1000t^2 + 2380t + 100$. For the actual cumulative demand by time *t*, let $D(t) = \int_0^4 d(x)dx = 4506.7 \cong 4507$. Let the retail price (*P*) be \$200, the wholesale price (*C*) be \$120, the production cost per unit (C_V) be \$60, the buyer's salvage value (S_b) be \$40, the supplier's salvage value (S_S) be \$10, the overstock cost to the buyer ($C_1^O = C - S_b$) be \$80, the understock cost to the buyer ($C_1^U = P - C$) be \$80, the overstock cost to supplier *i* ($C_i^O = C_V - S_S$) be \$50, and the understock cost to supplier *i* ($C_i^U = C - C_V$) be \$60, respectively, $D_0 = 100$, $F_i^0 = 105$, $G_i^0 = \left(K_i^0\right)^2 + 50$, and $\sigma_i = 50$, for *i* = *1*, *2*, *3*, *4*, and *5*. Suppose the demand forecast and the actual demand are the same. In that case, the profits before accounting for the forecasting costs are estimated as follows: \$360,533 for the buyer, \$270,400 for supplier *i*, and \$630,933 for the coordinated supply chain.

4.1. The Expected Profit (Π_1^a) for the Buyer

By using (2), we evaluate his expected profit (Π_1^a) under different combinations of α_1 and σ_1. Let $\alpha_1 \in \{10, 20, 30\}$ and $\sigma_1 \in \{10, 50, 100\}$. We also measure the buyer's forecasting errors resulting from his FC and the forecasting volatility by the monetary value, which is the difference between the expected profit (Π_1^a) and the profit when $D(t) = F(t)$. Table 1 summarizes the buyer's expected profit, and Fig.1 shows the buyer's forecasting error in terms of the dollar amount. Fig.1 indicates that the error amount decreases as the forecasting volatility reduces and that the buyer can curtail the error amount effectively by improving his FC. For example, the lowest error amount in this numerical example is estimated at \$94 when $\alpha_1 = 30$ and $\sigma_1 = 10$, whereas the highest error amount is at \$2,431 when $\alpha_1 = 10$ and $\sigma_1 = 100$.

(α_1, σ_1)	(10,100)	(20,100)	(30,100)	(10,50)	(20,50)	(30,50)	(10,10)	(20,10)	(30,10)
Π_1^a	\$358,102	\$358,509	\$358,809	\$359,595	\$359,632	\$359,732	\$360,305	\$360,434	\$360,439

Table 1. The buyer's expected profit.

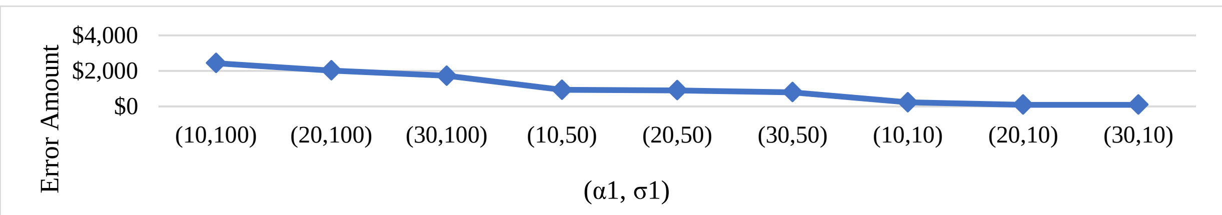

Fig. 1. The buyer's error amount based on (α_1, σ_1) where $\alpha_1 \in \{10, 20, 30\}$, $\sigma_1 \in \{10, 50, 100\}$.

4.2. *The Expected Profit (Π_i^a) for Supplier i*

The supplier's forecasting performance can be influenced by her own FC, her buyer's FC, forecasting volatility, her trust level in the buyer, and the buyer's trust level in her. To quantify supplier *i*'s profitability, we utilize different FC and partner trust levels with fixed forecasting volatility. Let $\alpha_1 \in \{15, 30\}$, $\alpha_i \in \{15, 30\}$, and $\sigma_i = 50$, for *i = 2, 3,* and *4*.

4.2.1. Supplier 2's Expected Profit (Π_2^a):

Considering that supplier *2*'s performance can be affected by the buyer's FC, we estimate her expected profit (Π_2^a) by using (3) and summarize them in Table 2. The maximum expected profit for supplier *2*, for example, is estimated at $269,467 with the projected error amount of $933 when the FCs of the buyer and supplier *2* are the highest (i.e., α_1 = 30 and α_2 = 30), whereas her minimum expected profit turns out to be $268,710 with the projected error amount of $1,689 when both parties' FCs are the lowest (i.e., α_1 = 15 and α_2 = 15).

(α_1, α_2)	(15,15)	(15,30)	(30,15)	(30,30)
Π_2^a	$268,710.38	$268,938.60	$269,375.23	$269,466.88
Error amount	$1,689.62	$1,461.40	$1,024.77	$933.12

Table 2. The expected profit for supplier 2.

4.2.2. Supplier 3's Expected Profit (Π_3^a):

In the buyer-supplier *3* relationship, we consider the role of partner trust level (r_i) with supplier dependence (w_i) and analyze how the two variables can influence the profitability of supplier *3*. Let $\alpha_1 = \alpha_3 = 15$ and $w_1 \in \{0.7, 0.3\}$. To clarify how different levels of partner trust can affect her expected profit (Π_3^a) at time *t*, we adopt two forms of trust level for the buyer and the supplier, respectively. Let $r_1(t) = 1.0$, $r_1(t) = -0.05t^2 + 0.25t + 0.6875$, $r_3(t) = 1.0$, $r_3(t) = -0.0397t^2 + 0.214t + 0.7091$, $r_{13}^T = f(r_1) = \{2 - r_1(t)\}$, and $r_{31}^T = f(r_3, r_{13}^T) = \{2 - r_1(t)\}r_3(t)$. We calculate the expected profit using (3) under different combinations of r_i and w_i and summarize them in Table 3. Supplier *3* obtains her highest expected profit with the least amount of forecasting error when she and the buyer trust each other by the absolute level throughout their working relationship such that $r_1(t) = r_3(t) = 1.0$ for all $t \in [0, 4]$

$r_1(t)$	$r_3(t)$	Π_3^a with w_1=0.7	Error Amount	Π_3^a with w_1=0.3	Error Amount
$-0.05t^2 + 0.25t + 0.6875$	$-0.0397t^2 + 0.214t + 0.7091$	$265,685.68	$4,714.32	$266,158.16	$4,241.84
$-0.05t^2 + 0.25t + 0.6875$	1	$237,233.22	$33,166.78	$261605.70	$8,794.30
1	$-0.0397t^2 + 0.214t + 0.7091$	$250,480.67	$19,919.33	$265548.83	$4,851.17
1	1	$269,069.49	$1,330.51	$268990.65	$1,409.35

Table 3. The expected profit for supplier 3.

4.2.3. Supplier 4's Expected Profit (Π_4^a):

In estimating supplier *4*'s expected profit, we use the same numerical parameters for the trust level and the FC as those used in the section of supplier *3*'s expected profit. By using (3), we evaluate her expected profit (Π_4^a) and summarize them in Table 4. Just as shown in Table 3, supplier *4* also can achieve her highest expected profit with the lowest amount of forecasting error when both parties maintain an absolute mutual trust level during their working relationship.

$r_1(t)$	$r_4(t)$	Π_4^a	Error Amount
$-0.05t^2 + 0.25t + 0.6875$	$-0.0397t^2 + 0.214t + 0.7091$	$261,331.80	$9,068.20
$-0.05t^2 + 0.25t + 0.6875$	1	$220,528.76	$49,871.24
1	$-0.0397t^2 + 0.214t + 0.7091$	$237,370.50	$33,029.50
1	1	$269,105.98	$1,294.02

Table 4. The expected profit for supplier 4.

4.3. The Expected Profit (Π_5^{ia}) for the Coordinated Supply Chain

We quantify the coordinated supply chain's expected profit (Π_5^{ia}) based on the condition of $\alpha_5^i(t) = f(r_1, \alpha_1, r_i, \alpha_i)$ for $i = 2$, 3, and 4. Let $\alpha_5^i \in \{10, 15, 20, 25, 30, 35\}$. Using (4), we estimate Π_5^{ia} and summarize them in Table 5. It shows that the supply chain's profitability increases with a decreasing error amount as the FC improves. To examine in depth the impact of the trust-level factor of the combined FC on supply chain profitability, we measure Π_5^{ia} based on different combinations of partner trust levels (r_1 and r_i) with fixed individual FC ($\alpha_1 = \alpha_i$) and summarize the results in Table 6. Let $(r_1, r_i) \in \{0.5, 0.8, 0.9, 1.0\}$, $\alpha_1 = 15$, $\alpha_i = 15$, and $\alpha_5^i(\alpha_1, r_1, \alpha_i, r_i) = \left(\frac{\alpha_1 r_1 + \alpha_i r_i}{2}\right)$. The outcomes in Table 6 show that if the combined FC is an increasing function of both r_1 and r_i, the maximum supply chain profit occurs at both $r_1 = 1.0$ and $r_i = 1.0$. We find that the results of Tables 5 and 6 support Theorem1. To investigate the coordinated supply chain's profit when its combined FC is concave on trust levels, we set the joint FC as $\alpha_5^i(t) = \left\{\frac{\alpha_1(1.8r_1 - r_1^2) + \alpha_i(2r_i - r_i^2)}{2}\right\}$, where $\alpha_1 = \alpha_i = 15$. Using this concave FC parameter, we calculate Π_5^{ia} based on different combinations of r_1 and r_i and summarize them in Table 7. As demonstrated in Theorem 2, Table 7 indicates that the maximum profit of the coordinated supply.

α_5^i	10	15	20	25	30	35
Π_5^{ia}	\$629,366	\$629,684	\$629,847	\$629,955	\$630,036	\$630,099
Error Amount	\$1,567	\$1,249	\$1,087	\$978	\$897	\$834

Table 5. The coordination supply chain profit under α_5^i.

r_1	r_i	α_5^i	Π_5^{ia}
0.5	0.5	7.5	\$628,961
0.5	0.9	10.5	\$629,414
0.5	1	11.25	\$629,476
0.8	0.8	12	\$629,529
0.8	0.9	12.75	\$629,575
0.8	1	13.5	\$629,615
0.9	1	14.25	\$629,651
1	1	15	\$629,684
$-0.05t^2 + 0.25t + 0.6875$	$-0.0397t^2 + 0.214t + 0.7091$	----	\$629,647

Table 6. The coordinated supply chain profit under $\alpha_5^i(\alpha_1, r_1, \alpha_i, r_i) = \left(\frac{\alpha_1 r_1 + \alpha_i r_i}{2}\right)$.

(r_1, r_i)	(0.5, 0.6)	(0.6, 0.7)	(0.7, 0.8)	(0.8, 0.9)	(0.9, 1.0)	(1.0, 1.0)
Π_5^{ia}	\$629,680	\$629,734	\$629,768	\$629,787	\$629,794	\$629,790

Table 7. The coordinated supply chain profit under the concave function of $\alpha_5^i(t)$.

4.3.1. Forecasting Cost due to Collaborative Forecasting Coordination:

Combining the buyer's and the supplier's FC to form a joint working system may require additional investments in technology, systems, and other areas to build and maintain a trusting relationship between them. In this section, we evaluate how the additional forecasting expenses (C_i^{CF}) can affect supply chain profitability. For the coordinated supply chain to be more profitable than the one with no collaboration, there must be $C_i^{CFM} < \Pi_5^{iaP}$. We highlight that the estimation of Π_5^{iaP} provides the maximum limit of the additional forecasting cost for the coordinated supply chain. Let $r_1 = 1.0$, $r_2 = 1.0$, and $\alpha_5^2 = \left(\frac{\alpha_1 r_1 + \alpha_2 r_2}{2}\right)$. Table 8 shows supplier *2*'s additional profit (Π_5^{2aP}) when $\Pi_5^2 > \Pi_1 + \Pi_2$ under different combinations of supply chain partner FCs. Based on the given set of $\alpha_1 = 15$ and $\alpha_2 = 15$, for example, Π_5^{2aP} is estimated at \$1,184.55. This result implies that the profitability of the joint forecasting supply chain is higher than that of the no-collaboration supply chain if its maximum additional cost (C_2^{CFM}) is less than \$1,184.55. For the buyer-supplier *3* relationship, let $\alpha_i \in \{15, 30\}$, $r_i \in \{0.7, 1.0\}$, $\alpha_5^3 = \left(\frac{r_1\alpha_1 + r_3\alpha_3}{2}\right)$, and $w_1 = 0.7$. For the buyer-supplier *4* relationship, let $\alpha_i \in \{15, 30\}$, $r_i \in \{0.7, 1.0\}$, and $\alpha_5^4 = \left(\frac{r_1\alpha_1 + r_4\alpha_4}{2}\right)$. Table 9 summarizes supplier *3*'s additional profit (Π_5^{3aP}) when $\Pi_5^3 > \Pi_1 + \Pi_3$ and Table 10 summarizes supplier *4*'s additional profit (Π_5^{4aP}) when $\Pi_5^4 > \Pi_1 + \Pi_4$ under different combinations of supply chain partner FC, respectively. As shown in Tables 9 and 10, if $C_i^{CFM} < \Pi_5^{iaP}$, the expected supply chain profit for both cases (Π_5^3 and Π_5^4) is also higher than that of the no-collaboration supply chain. If α_5^i is not a concave function, the supply chain profit can be maximized when the buyer and his supplier display their partner trust at an absolute level such that $r_1 = r_3 = 1.0$ in Table 9 and $r_1 = r_4 = 1.0$ in Table 10. All the results of Tables from 8 to 10 support Corollary 1 and 2.

α_1	α_2	α_5^2	Π_1^a	Π_2^a	Π_5^{2a}	Π_5^{2aP}
15	15	15	$359,589.18	$268,910.38	$629,684.11	$1,184.55
15	30	22.5	$359,589.18	$268,938.60	$629,905.43	$1,377.65
30	15	22.5	$359,731.57	$269,375.33	$629,905.43	$798.53
30	30	30	$359,731.57	$269,466.88	$630,036.06	$837.61

Table 8. The expected additional profit in the buyer-supplier 2 relationship.

α_1	α_3	r_1	r_3	α_5^3	Π_1^a	Π_3^a	Π_5^{3a}	Π_5^{3aP}
15	15	0.7	0.7	10.5	$359,589.18	$238,128.67	$629,414.22	$31,696.37
15	15	1	1	15.0	$359,589.18	$269,069.49	$629,684.00	$1,025.33
30	30	0.7	0.7	28.0	$359,731.57	$236,072.87	$630,006.28	$34,201.84
30	30	1	1	30.0	$359,731.57	$269,517.60	$630,036.00	$786.83

Table 9. The expected additional profit in the buyer-supplier 3 relationship.

α_1	α_4	r_1	r_4	α_5^4	Π_1^a	Π_4^a	Π_5^{4a}	Π_5^{4aP}
15	15	0.7	0.7	10.5	$359,589.18	$221,127.73	$629,414.22	$48,697.31
15	15	1	1	15.0	$359,589.18	$269,105.98	$629,684.00	$988.84
30	30	0.7	0.7	28.0	$359,731.57	$221,373.41	$630,006.28	$48,901.30
30	30	1	1	30.0	$359,731.57	$269,593.74	$630,036.00	$710.69

Table 10. The expected additional profit in the buyer-supplier 4 relationship.

Our numerical analysis results show that demand forecast sharing supported by a higher level of partner trust can positively affect the supply chain performance outcomes. They also indicate that the coordinated supply chain can maximize its expected profit (Π_5^i) by increasing partner trust levels (r_i) for both the buyer and his supplier to an absolute (r_i = 1.0).

5. CONCLUSION

This paper studies the role of partner trust level in a collaborative demand forecast sharing process under a high demand-volatile supply chain with a single buyer (*i = 1*) and a single supplier of three types (*i = 2, 3,* and *4*). We propose an analytical model to evaluate the contribution of partner trust level to the forecasting coordinated supply chain (*i* = 5) and determine the optimal level of partner trust that the buyer and his supplier *i* should retain to achieve the coordinated supply chain's maximum profitability.

The main results of our study are threefold. First, if the combined FC (α_5^i) increases by enhancing the trust levels of the buyer and his supplier (r_1 and r_i), the expected profit of the supply chain (Π_5^{ia}) can also grow with the increased levels of partner trust and is maximized when both parties hold their trust levels as an absolute – that is, r_1 = 1.0 and r_i = 1.0. We assume that this is because the buyer and his supplier *i* believe that their trading partner will not behave opportunistically and will be willing to share credible demand forecasts, including private forecast information, as discussed in some research papers (Cheng et al., 2008; Fawcett et al., 2012; Ebrahim-Khanjari et al., 2012; Ali et al., 2012; Hyndman et al., 2013; Gao, 2015)Second, the coordinated supply chain's expected profit is always higher than that of the supply chain without forecasting collaboration as far as the profit estimate (Π_5^{iaP}) outweighs the additional forecasting expenses (C_i^{CF}). Furthermore, as the marginal additional profit rate ($\partial\Pi_5^{iaP}/\partial\alpha_5^i$) increases faster than the marginal additional cost rate ($\partial C_i^{CF}/\partial\alpha_5^i$), the expected additional net profit (Π_5^{inP}) is maximized when both the buyer and his supplier *i* retain the absolute level of partner trust throughout their working relationship. As shown in our numerical examples, regardless of the supplier types, the coordinated supply chain profit (Π_5^i) is higher than the sum of the two individual partners' profits attained without demand forecast sharing ($\Pi_1 + \Pi_i$, for *i = 2, 3,* and *4*). Third, if the combined FC is a concave function of trust level with $\lambda_i > 0$, the coordinated supply chain profit is maximized at $r_{1i}^* = (r_1^*, r_i^*)$. The supply chain partners, therefore, need to investigate which level of r_{1i} can generate the highest level of α_5^i.

Our findings reveal some important managerial implications. The enhancement of partner trust level helps increases the FC of a supply chain and its profitability under a collaborative demand forecast sharing scheme. The improvement in individual FC undertaken by the buyer and his supplier *i* may promote supply chain surplus by reducing forecasting volatility. However, from a supply chain management standpoint, rather than putting additional money and resources to the advancement of individual partners' FC, enhancing trust level by plainly exchanging truthful information and sharing a common demand forecast can be a more cost-efficient method in improving supply chain performance. To establish a trustful buyer-supplier relationship, the two parties need to understand how the forecasting collaboration can yield a productive joint effect that is greater in value than the individual partners could create by predicting a demand forecast separately. Once the partners agree to engage in demand forecast sharing with a joint goal of profit maximization, they should be able to hold a collaborative trust that is as closer to an absolute level as possible throughout their working relationship.

The model developed in this study can be used as a decision-making tool for firms in the supply chain to select the optimum trust-level policy that can maximize the economic payoff from collaborative demand forecast sharing. Our model provides the framework that enables the buyer and supplier i in the coordinated supply chain to assess the expected additional profit (Π_5^{iaP}) with a maximum limit of the additional forecasting cost (C_i^{CFM}) based on various combinations of partner trust levels (r_1 and r_i) so that they can analyze how the expected additional net profit (Π_5^{inP}) can change as they modify their trust levels.

As to limitations, our model was developed based on a single buyer and a single supplier in the supply chain. Despite the multidimensionality of supply chain trust, we focused mainly on its capability- and risk-based components. We also used a single-period newsvendor framework to capture the impact of different partner trust levels on collaborative forecasting performance and supply chain profitability. It would be useful to examine an expanded supply chain structure's effect on this study's conclusion by adopting multiple buyers and suppliers with long-term interaction relationships. Future research also needs to quantify each partner's share of the total expected supply chain profit to offer a basis for a demand forecast sharing decision. In the context of joint forecasting coordination, understanding how overstock and understock penalty costs can influence the determination of the optimal trust-level policy is also worthy of further investigation.

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