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Informed Trading in Multi-Asset Markets

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Abstract

A model of trading multiple assets in a financial market by a risk neutral insider is employed to examine the price formation and the insider's trading behavior. Due to information spillover across different assets, the insider strategically trades the multiple assets in order to maintain the information structure across the assets. As a result, trading intensities and market depths in one asset are closely intertwined with those in other assets. The insider may trade very intensely even with low asset correlations

Keywords: Informed trading, Multi-asset securities, Correlations, Private information, Liquidity

1. Introduction

In a seminal article, Kyle (1985) investigated how an informed trader strategically makes use of his private information in an asset market. His paper shows that the insider has incentives to trade slowly so as not to reveal too much information. In addition, as the number of trading periods increases, information is incorporated in the price at a constant rate and the depth of the market is also constant. However, Kyle (1985), together with many extensions of his work, focused on only one asset.[1] It is unclear whether similar results can be obtained in multi-asset securities trading. In practice, traders may trade multiple assets simultaneously in an actual financial market. Thus, it is sensible that an insider's trading strategy may include simultaneous trading in multiple assets.

The main difference between a single-asset market setting and a multiasset setting lies in information spillover across asset securities markets. As it turns out, the correlations between assets play a dual role in a multi-asset trading setting.[2] On the one hand, the correlations allow the uninformed to learn additional information about unknown asset payoffs from all observed orders as each observed order could potentially contain information about all asset payoffs. On the other hand, to limit the information revealed from the observed orders to others, the insider must adjust the orders in all assets based on correlations between the assets. It is this type of information transmission and revelation across assets that we try to capture in our model and use it to investigate how the correlations between different assets affect the insider's trading behavior and price formation.

The multiplicity of assets makes it possible to address issues that cannot even be formulated in models within a single-asset framework.[3] For instance, how does information spillover across markets affect the insider's trading behavior in each asset and how do asset prices change with the insider's order(s)? How does the insider's profit in each asset change with asset correlations?

The major studies of multi-asset trading in financial markets include Admati (1985), Caballe & Krishnan (1994) and Bernhardt & Taub (2008).[4] Among them, Admati (1985) introduced a multi-asset, noisy rational expectations model with a continuum of agents. Under general covariance structures, Caballe & Krishnan (1994) derived an equilibrium of imperfectly competitive trading in a multi-security market. Bernhardt & Taub (2008) developed a strategic analogue of the noise rational expectations equilibrium model of Admati (1985) by assuming that informed traders not only know pricing rules but also know actual prices even before their orders are submitted or executed.[5] This, however, contradicts the essence of a setting with market orders in real financial markets, where traders are unable to either observe or correctly anticipate actual prices when submitting orders.

Due to the matrix structure inherent in multi-asset settings, it becomes formidable to keep track of various complex interactive effects under general conditions. One of the main contributions of this paper is to provide a solution to tracking the various interactive effects within a multi-asset trading framework under a relatively rich set of conditions. This is important both for the analysis of various aspects of trading in financial markets and for producing specific and empirically testable results. The second main contribution, which comes from analysis of the solution, shows that additional new insights can be obtained about how information spillover across assets affects multi-asset trading and price formation.

Our analysis shows that the information spillover across assets has a substantial impact on equilibrium prices and the insider's trading behavior. On the one hand, the equilibrium price determined for each asset conveys information from two different sources: the order information from this particular asset market, and the information revealed from the orders in the other assets markets. Each source of information affects the price differently, and the weight carried by each source is, of course, dependent on the correlation structure of the assets. Intuitively, the more correlated the assets, the less weight carried by the first source and the more weight carried by the second source. On the other hand, the insider must adjust his order in each asset market to prevent from revealing too much information to others. As a result, even with low correlations, the insider may still trade very intensively in multiple assets. By doing so, the insider balances profits across assets and across time: profit from trading in one asset against the reduction of profits from trading in other assets. Not surprisingly, our findings suggest that the correlations across assets reduce the insider's expected profits: the insider earns relatively less expected profit per asset, compared to trading in only one asset market.

The paper proceeds as follows. The model is described in Section 2. Section 3 presents our analysis followed by numerical demonstrations. Concluding remarks are given in Section 4. Technical proofs are provided in the Appendix.

2. The Model

The following model is a multi-asset generalization of Kyle (1985). In the model, three types of traders, including a market maker, an insider, and a number of liquidity traders, buy and sell M different securities over N periods. At the beginning of the first period, the insider observes the liquidation values of all securities. Based on this information, the insider places orders with the market maker. At each period the market maker sees the combined order flows from the insider and liquidity traders in each asset, and then determines prices for all M assets. At the end of the last period, the liquidation values of all assets are announced.

Specifically, we denote the vector of ex post liquidation values of M securities at the end of trading as $v = (v_1,...,v_M)'$, which is assumed to be normally distributed: $v \sim \mathrm{N}(p_0,\mathbf{\Sigma}_0)$, with $p_0 = (p_{10},...,p_{M0})'$ and $\mathbf{\Sigma}_0$ being an $M \times M$ positive-definite variance-covariance matrix. Further, we allow correlation between v_i and v_j for $i \neq j$. But for ease of presentation of the main idea, we assume equal covariances, $\rho\sigma^2$ with $|\rho| \leq 1$, between v_i and v_j for $i \neq j$ and equal variances, σ^2, for v_i, $i,j = 1,...,M$.[6] Notice that due to the positive definiteness of the matrix, the parameter ρ can only take values between $-\frac{1}{M-1}$ and 1. Our aim is to investigate how the price formation and the insider's trading behavior are affected by the changing values of ρ.

At the nth period, $n = 1,...,N$, let the $M \times 1$ vector x_n denote the quantity traded by the insider in all M assets, and the $M\times 1$ vector u_n denote the quantity traded by all liquidity traders. We assume liquidity traders trade independently across assets and the quantity traded in each asset has a zero mean normal distribution with variance σ_u^2. That is, $u_n \sim \mathrm{N}(\mathbf{0},\sigma_u^2\mathbf{I}_M)$, where $\mathbf{I}_M$ is the identity matrix with dimension M. Additionally, u_n is assumed to be independent of v. At each period, the market maker receives orders from the insider and noise traders, but he can only observe the combined orders $y_n = x_n + u_n$. Then based on his observed information, the market maker determines the price p_n to clear all M asset markets.

Let X_n denote the insider's trading strategy and P_n denote the market maker's pricing rule at the nth trading period. We have $x_n = X_n(p_1,...,p_{n-1},v)$ for $n = 1,...,N$ and $p_n = P_n(y_1,...,y_n)$ for $n = 1,...,N$. That is, x_n is a $M \times 1$ vector which depends on the liquidation value vector v and past prices of all assets, and p_n is a $M \times 1$ vector which depends on the current total order flow y_n and all past total order flows. Notice that all M assets are related to each other in the sense that the price determined in each asset depends on the orders submitted in all M assets. Similarly, the order submitted by the insider in each asset depends on the liquidation values and market prices of all M assets.

The insider is assumed to maximize his total expected profits from trading all M assets in the future. Let π_n denote the insider's total profits from future trading periods $n,...,N$. Clearly, $\pi_n = \sum_{k=n}^{N}(\boldsymbol{v} - \boldsymbol{p}_k)'\boldsymbol{x}_k$.

Following Kyle (1985), we define the sequence of the insider's trading strategies and the market maker's pricing rules X and P as $X \equiv \langle X_1,...,X_N \rangle$ and $P \equiv \langle P_1,...,P_N \rangle$, respectively. In a sequential equilibrium, X and P must satisfy the following two conditions:

1. For all $n = 1,...,N$, and $X^* = \langle X_1^*, \ldots, X_N^* \rangle$ such that $X_1^* = X_1, \ldots, X_{n-1}^* = X_{n-1}$, we have

$$\boldsymbol{E[\pi_n(X,P)|p_1,...,p_{n-1},v] \geq E[\pi_n(X^*,P)|p_1,...,p_{n-1},v]}.$$

2. For all $n = 1,...,N$, we have

$$\boldsymbol{p_n = E[v|y_1,...,y_n]}.$$

In a linear equilibrium, x_n and p_n must take the following forms:

$$\begin{aligned} \boldsymbol{X_n} &= \boldsymbol{\Gamma_n(v - p_{n-1})}, \\ \boldsymbol{P_n} &= \boldsymbol{p_{n-1} + \Lambda_n(x_n + u_n)}, \end{aligned}$$

where both $\mathbf{\Gamma}_n$ and $\mathbf{\Lambda}_n$ are $M \times M$ constant matrices. Denote the elements of $\mathbf{\Gamma}_n$ and $\mathbf{\Lambda}_n$ as (β_{ij}) (λ_{ij}), $i, j = 1,...,M$ respectively. Obviously, for any given n, $n = 1,...,N$, each element of x_n is a linear form of $v_j - p_{jn-1}$, and each element of p_n is a linear form of $x_{jn} + u_{jn}$, for $j = 1,...,M$. To be specific, for the ith element of x_n and p_n, $i = 1,...,M$, $n = 1,...,N$, we have

$$x_{in} = \sum_{j=1}^{M} \beta_{ijn}(v_j - p_{jn-1}), \quad (1)$$
$$p_{in} = P_{in-1} + \sum_{j=1}^{M} \lambda_{ijn}(x_{jn} + u_{jn}). \quad (2)$$

These linear representations indicate that the order submitted by the insider in each asset conveys information from all M assets, and the price set by the market maker in each asset contains the insider's order information from all M assets. As shown in next section, both equations (1) and (2) demonstrate the main difference between the multi-asset setting and the single asset market setting. Furthermore, in next section, we will also conduct an in-depth analysis of the slopes in (1) and (2), β_{ijn} and λ_{ijn}, $i,j = 1,...,M$, which measure the impact of the information from each asset j on x_{in} and p_{in}, respectively.

3. Analysis

To introduce the main idea and motivate the understanding of the insider's trading behavior in multiple securities, we discuss a simple one-period model in this section. Since there is only one trading period, the subscript n is suppressed without causing confusion in this section. The following result is a generalization of the single-auction equilibrium in Kyle (1985) to multiple securities.

Proposition 3.1

If $-\frac{1}{M-1} < \rho < 1$, *there is a unique linear symmetric equilibrium in which*

$$\boldsymbol{x} = \boldsymbol{\Gamma}(\boldsymbol{v} - \boldsymbol{p_0}), \quad (3)$$
$$\boldsymbol{p} = \boldsymbol{p_0} + \boldsymbol{\Lambda}(\boldsymbol{x} + \boldsymbol{u}), \quad (4)$$

where $\boldsymbol{\Gamma} = \sigma_u \boldsymbol{\Sigma}_0^{-\frac{1}{2}}$ *and* $\boldsymbol{\Lambda} = \frac{1}{2\sigma_u}\boldsymbol{\Sigma}_0^{\frac{1}{2}}$. *Equivalently, the ith elements of x and p,* x_i *and* p_i, $i = 1,...,M$, *take the following forms*

$$x_i = \beta_1(v_i - p_{i0}) + \beta_2 \Sigma_{j \neq i} (v_j - p_{j0}), \quad (5)$$
$$p_i = p_{i0} + \lambda_1(x_i + u_i) + \lambda_2 \Sigma_{j \neq i} (x_j + u_j), \quad (6)$$

where

$$\beta_1 = \left[\frac{1}{M}\frac{1}{\sqrt{1+(M-1)\rho}} + \left(1-\frac{1}{M}\right)\frac{1}{\sqrt{1-\rho}}\right]\frac{\sigma_u}{\sigma} \quad (7)$$
$$\beta_2 = \frac{1}{M}\left[\frac{1}{\sqrt{1+(M-1)\rho}} - \frac{1}{\sqrt{1-\rho}}\right]\frac{\sigma_u}{\sigma}, \quad (8)$$
$$\lambda_1 = \left[\frac{1}{M}\sqrt{1+(M-1)\rho} + \left(1-\frac{1}{M}\right)\sqrt{1-\rho}\right]\frac{\sigma}{2\sigma_u}, \quad (9)$$
$$\lambda_2 = \frac{1}{M}\left[\sqrt{1+(M-1)\rho} - \sqrt{1-\rho}\right]\frac{\sigma}{2\sigma_u}. \quad (10)$$

Also,

$$E[\pi] = \frac{1}{2}\sigma_u\sigma\left(\sqrt{1+(M-1)\rho} + (M-1)\sqrt{1-\rho}\right). \quad (11)$$

PROOF: See Appendix A.1.

Comparing the above equilibrium with the one of the single-asset models in the literature such as Kyle (1985), we can easily see similarities and differences between the two. Similar to the single-asset models , Proposition 3.1 shows that, for the ith asset, its price function (6) depends on the asset's own orders submitted, $x_i + u_i$, through λ_1, and the order submitted by the insider in the asset (5) depends on the the asset's own payoff information, $v_i - p_{i0}$, through β_1. But different from the single-asset models, the Proposition shows the ith asset's price function and the order submitted by the insider in the ith asset also include additional terms: $\beta_2 \Sigma_{j \neq i} (v_j - p_{j0})$ in (5) and $\lambda_2 \Sigma_{j \neq i} (x_j + u_j)$ in (6). The extra components $\Sigma_{j \neq i} (v_j - p_{j0})$, containing payoff information of the rest $M - 1$ assets, affect the order submitted by the insider in the ith asset through β_2; the extra components $\Sigma_{j \neq i} (x_j + u_j)$, containing order information of the rest of $M - 1$ assets, affect the ith asset price through λ_2.

Evidently, the extra components in (5) and (6) demonstrate that, in a multi-asset securities market, there exists information spillover across assets. Other things equal, a change in one asset's payoff affects the orders submitted by the insider in all assets. Similarly, a change in one asset's order affects the prices of all assets. In other words, any information about an asset is used to learn not only about the asset itself but also about other related assets. On the one hand, the market maker tries to learn the liquidation values from all the orders submitted in all assets. On the other hand, the insider, who tries to maximize the total expected profits from trading all M assets, must resist from revealing too much information by adjusting his order in each asset. By doing so, he balances the profit from trading in one asset against the reduction in profits from trading in the other assets.

How much information do the market prices reveal after the trading? How do asset correlations change after the trading? A simple calculation shows $\mathrm{Var}(\boldsymbol{v}|\boldsymbol{p}) = \frac{1}{2}\mathrm{Var}(\boldsymbol{v}) = \frac{1}{2}\boldsymbol{\Sigma}_0$. That is, similar to the one-asset model, half of the insider's private information is incorporated into prices. Furthermore, it shows the asset correlations do not change. Hence, we have the following result:

Proposition 3.2 ***In the above model, the conditional asset correlations after the trading are the same as the initial asset correlations.***

The above result demonstrates that, the insider optimally reveals his private information about the multiple assets - neither too much nor too less. This is just the direct result of the insider's profit balancing from trading one asset against another, as mentioned before. In addition, the market also extracts information about the multiple assets.

Although any information change in a multi-asset market has impact on all assets, the impact is not the same across the assets. Any impact can be measured through the four parameters in Proposition 3.1. Among them, λ_1 measures the marginal trading cost (the depth of the market with a small λ corresponding to a deep market) and β_1 measures the intensity with which the insider trades on his private information about any asset itself, as in the single-asset setting. In addition, the above multi-asset environment also introduces two new parameters, λ_2 and β_2, where λ_2 measures the marginal trading cost and β_2 measures the trading intensity across assets.

It is interesting to see, other things equal, a unit increase of liquidity orders in one asset, not only increases the asset's own price by λ_1, but also increases the prices of the other assets by λ_2. In other words, the asset prices comove together (either in the same direction or in opposite directions depending on the signs of λ_1 and λ_2). Not surprisingly, both liquidity measures are proportional to σ/σ_u and both trading intensity measures are proportional to σ_u/σ. That is, increasing liquidity trading volatility creates more camouflage and hence increases both liquidity measures and trading intensity measures. Finally, in the above multi-asset securities market, all four parameters depend on the correlation ρ.

To further investigate how the correlations between the assets affect the price function and the insider's trading behavior, we examine how the values of the above four parameters and the insider's expected profits $E[\pi]$ given in Proposition 3.1 change with the correlation parameter ρ. For ease of comparison, we denote the values of $(\lambda_1,\lambda_2,\beta_1,\beta_2,E[\pi])$ at zero correlation, which corresponds to the single-asset case, as $(\lambda_{10},\lambda_{20},\beta_{10},\beta_{20},E[\pi]_0)$. In addition, we take the first order derivatives of $(\lambda_1,\lambda_2,\beta_1,\beta_2)$ with respect to ρ to find out how they change with the correlation. The results are summarized in the following proposition.

Proposition 3.3 ***In the above one-period model, we have***

1. $0 < \lambda_1 < \lambda_{10}$, $0 < \beta_1 < \beta_{10}$; $\lambda_{20} = \beta_{20} = 0$;
2. *(a) for* $\rho > 0$, $\lambda_2 > 0$, $\beta_2 < 0$, $\partial\lambda_1/\partial\rho < 0$, $\partial\beta_1/\partial\rho > 0$;

   (b) for $-1/(M-1) < \rho < 0$, $\lambda_2 < 0$, $\beta_2 > 0$, $\partial\lambda 1/\partial\rho > 0$, $\partial\beta 1/\partial\rho < 0$;

   (c) $\partial\lambda 2/\partial\rho > 0$, $\partial\beta 2/\partial\rho < 0$
3. $E[\pi] < E[\pi]_0$.

PROOF: See Appendix A.2.

The interpretations of Proposition 3.3 are the following. Part 1 says that, for any asset in a multi-asset setting, like the single-asset case, both its marginal trading cost and its trading intensity are always positive. But since the market maker also obtains additional information from the other assets in the multi-asset setting, the information weight from the ith asset is reduced. Hence $\lambda_1 < \lambda_{10}$. On the other hand, the insider, who tries to resist from revealing too much information, must use the information from the ith asset more aggressively and trade it relatively more intensively. As a result, $\beta_{10} < \beta_1$. In addition, as explained before, in the single-asset setting, the cross asset effect does not exist. Therefore, $\lambda_{20} = \beta_{20} = 0$. Also, λ_1 and β_1 become $\frac{\sigma}{2\sigma_u}$ and $\frac{\sigma_u}{\sigma}$, respectively, which are identical to Kyle (1985).

Part 2 says that, when assets are positively correlated, the market maker, expecting to get positively correlated information from the other assets, always assigns a positive weight to the order information from the other assets. Correspondingly, the insider, to offset related information from the other assets, always trades oppositely against the other assets. Thus $\lambda_2 > 0$, $\beta_2 < 0$ for $\rho > 0$. Both signs change to the opposite when $\rho < 0$. Furthermore, when assets are more either positively or negatively correlated (ρ becomes further away from zero), the market maker, expecting the information extracted from the other assets to increase, reduces the weight to the asset itself and increases the weight to the other assets. Correspondingly, the insider must increase the trading intensity against the *i*th asset and reduce the trading intensity against the other assets.

Part 3 says that, compared to the case of no correlation (single-asset setting), the existence of asset correlation in the multi-asset setting reduces the insider's expected profit in each asset. A simple calculation also shows that, the more correlated the assets (either positively or negatively), the less expected profit the insider makes.

Proposition 3.3 describes how the correlations affect the parameters through the first order derivatives. In addition, by taking the second order derivatives of the parameters with respect to the correlations, we can see how fast the parameters change with the correlations. It turns out additional insights about the insider's trading intensity parameters can be obtained.

Corollary 3.4 ***When ρ approaches to 1, β_1 approaches infinity and β_2 approaches to negative infinity; when ρ approaches to $-\frac{1}{M-1}$, both β_1 and β_2 approach to infinity.***

The result can be easily seen from (7) and (8), where $1 + (M - 1)$ and $1 - \rho$ are part of the denominators. While it is intuitive and easy to understand that the insider increases trading intensities when ρ approaches to 1, where the assets are nearly perfectly correlated, what about the case when ρ approaches $-\frac{1}{M-1}$, where asset correlations can be very low (when M increases)? The reason is because, in a multi-variate environment, even though the correlations between the variables are low, it is possible that the correlation between one variable and a linear combination of the rest variables can be very high. In other words, in a multi-asset environment, an asset can be highly correlated with a portfolio even though the asset is little correlated with any assets of the portfolio. As such, a multi-asset setting offers insights that can never be seen from a two-asset model. This can be illustrated through an example in a three-asset environment of the above model ($M = 3$). Imagine a portfolio consisting of a one-unit long position in each of any two assets. A simple calculation shows the correlation between this portfolio and the third asset is 2ρ, which means they become closely correlated when ρ approaches to -0.5 from above.[7] As a result, the insider, to prevent revealing too much information, must trade more intensively in all assets when ρ becomes closer to -0.5.

To better understand the above results about how the correlations affect the price formation and the insider's trading behavior, we draw the following two figures showing how the liquidity parameters and the trading intensity parameters change with the correlation ρ for $M = 3$.

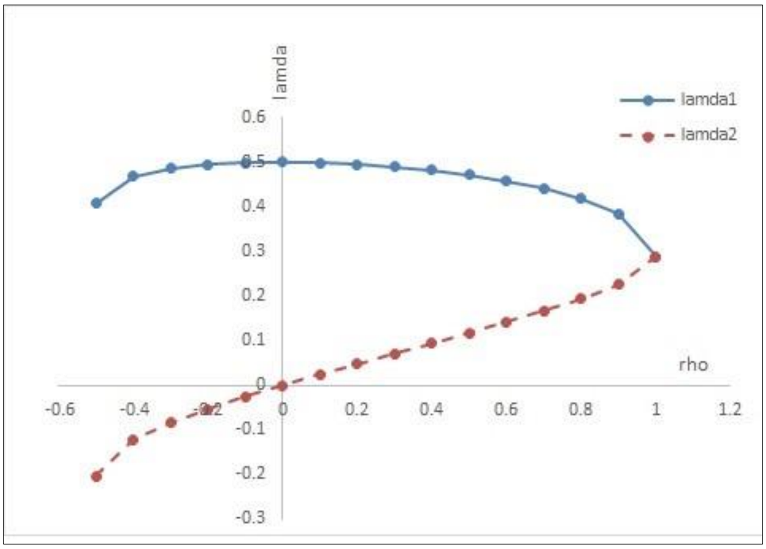

Figure 1: The change of liquidity parameters λ_1 and λ_2 with correlation values ρ (from −0.5 to 1) given M = 3, $\sigma = \sigma_u$ = 1. The solid line corresponds λ_1 and the dash line corresponds λ_2.

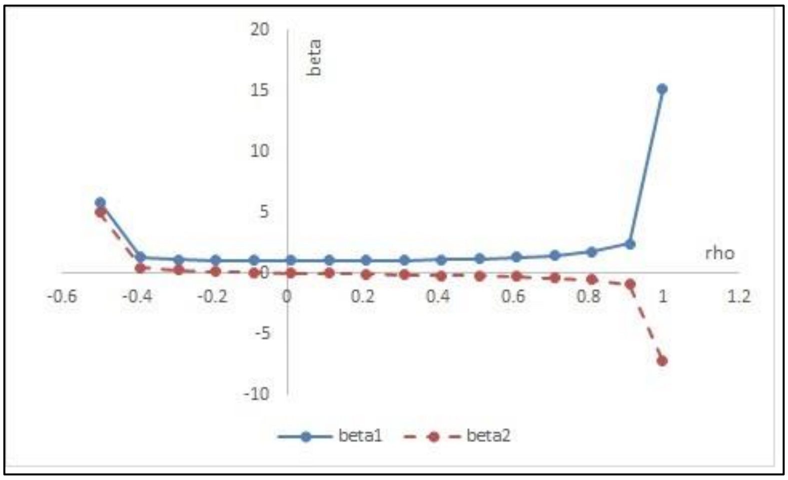

Figure 2: The change of trading intensity parameters β_1 and β_2 with correlation values ρ (from −0.5 to 1) given M = 3, $\sigma = \sigma_u = 1$. The solid line corresponds β_1 and the dash line corresponds β_2.

Figure 1 shows how the liquidity parameters λ_1 and λ_2 change with the correlation ρ for $M = 3$. It is obvious in this three-asset environment, any asset's own liquidity parameter decreases whenever there is nonzero correlation between the assets, but the cross asset liquidity parameter becomes either positive or negative, depending on the correlation. In addition, the cross liquidity parameter becomes more positive when ρ is more positive but more negative when ρ is more negative.

Figure 2 shows how the insider's trading intensity β_1 and β_2 change with the correlation ρ for $M = 3$. It can be seen that the trading intensity against any asset increases whenever there is nonzero correlation, and the insider trades against that asset more aggressively when the correlation is either more positive or more negative. Also, the insider trades in the opposite direction against the other assets when the correlation is positive but in the same direction when the correlation is negative. In addition, the insider trades more intensively against the other assets when the correlation becomes either more positive or more negative. Especially, notice that the insider increases all trading intensity more dramatically in magnitude when the correlation becomes closer to either 1 or −0.5.

Conclusion

Our analysis extends the Kyle (1985) model to a multi-asset securities market setting and considers correlations between assets in the model. Due to the information transmission and revelation across assets, the insider needs to adjust his order submitted in each asset, and the market maker also adjusts the price function in each asset. The correlations between assets significantly affect the insider's trading behavior and the formation of market prices. Even when there is not much correlation between the assets, the insider may still trade intensively in each asset. Also due to the information revelation across assets, the insider makes less expected profit in each asset market. Our findings indicate that information roles of market prices need to be considered across different asset markets and that the liquidity of different asset markets may be closely related to each other.

Appendix

A.1 Proof of Proposition 3.1

Suppose

$$\begin{aligned} \boldsymbol{P}(\boldsymbol{x}+\boldsymbol{u}) &= \boldsymbol{\mu} + \boldsymbol{\Lambda}(\boldsymbol{x}+\boldsymbol{u}), \\ \boldsymbol{X}(\boldsymbol{v}) &= \boldsymbol{\alpha} + \boldsymbol{\Gamma}\boldsymbol{v}, \end{aligned}$$

where both $\boldsymbol{\Gamma}$ and $\boldsymbol{\Lambda}$ are $M \times M$ matrices. Given the equal correlation and equal variance assumption of $\boldsymbol{\Sigma}_0$ in section 2, we conjecture both $\boldsymbol{\Gamma}$ and $\boldsymbol{\Lambda}$ are symmetric matrices, i.e., $\boldsymbol{\Gamma}' = \boldsymbol{\Gamma}$, $\boldsymbol{\Lambda}' = \boldsymbol{\Lambda}$. This conjecture is later confirmed to be correct by the results. Hence, in the proof below, $\boldsymbol{\Gamma}'$ and $\boldsymbol{\Gamma}$ are used interchangeably, so are $\boldsymbol{\Lambda}'$ and $\boldsymbol{\Lambda}$.

The insider's expected profit is $E[(v-p)'x|v] = [v - \mu - \boldsymbol{\Lambda}x]'x$. The first order condition

$$\boldsymbol{v} - \boldsymbol{\mu} - 2\boldsymbol{\Lambda}\boldsymbol{x} = \boldsymbol{0}$$

yields

$$\boldsymbol{x} = 1/2\, \boldsymbol{\Lambda}^{-1}(\boldsymbol{v} - \boldsymbol{\mu})$$

subject to the second order condition that $\mathbf{\Lambda}$ needs to be positive definite. Hence,

$$\frac{1}{2}\mathbf{\Lambda}^{-1} = \mathbf{\Gamma}, \qquad -\frac{1}{2}\mathbf{\Lambda}^{-1}\boldsymbol{\mu} = \boldsymbol{\alpha}. \tag{A1}$$

In addition, the market efficiency condition $p = E[v|x + u]$ implies

$$\mu = p_0,$$
$$\mathbf{\Lambda} = [(2\mathbf{\Lambda})^{-1} + \sigma_u^2(2\mathbf{\Lambda})\mathbf{\Sigma}_0^{-1}]^{-1}.$$

Inverting the last equation yields

$$\mathbf{\Lambda}^{-1} = (2\mathbf{\Lambda})^{-1} + 2\sigma_u^2\mathbf{\Lambda}\mathbf{\Sigma}_0^{-1},$$

or

$$\mathbf{\Lambda}^{-2} = 4\sigma_u^2\mathbf{\Sigma}_0^{-1}.$$

If $\mathbf{\Lambda}$ is symmetric, it must be positive definite and is uniquely defined as

$\mathbf{\Lambda} = \frac{1}{2\sigma_u}\mathbf{\Sigma}_0^{\frac{1}{2}}$. Plugging into (A1) yields $\mathbf{\Gamma} = \sigma_u\mathbf{\Sigma}_0^{-\frac{1}{2}}$ and $\boldsymbol{\alpha} = -\sigma_u\mathbf{\Sigma}_0^{-\frac{1}{2}}\boldsymbol{p}_0$.
Furthermore, the positive definiteness of $\mathbf{\Sigma}_0$ implies that $1 + (M - 1)\rho > 0$ and $1 - \rho > 0$, or equivalently, $-\frac{1}{M-1} < \rho < 1$. Hence we have (3) and (4).

In addition, all diagonal elements of $\mathbf{\Sigma}_0^{\frac{1}{2}}$ are

$$\left[\frac{1}{M}\sqrt{1+(M-1)\rho} + \left(1-\frac{1}{M}\right)\sqrt{1-\rho}\right]\sigma,$$

and all off-diagonal elements are

$$\frac{1}{M}\left[\sqrt{1+(M-1)\rho} - \sqrt{1-\rho}\right]\sigma.$$

Hence, we have (6) with λ_1 given in (9) and λ_2 given in (10). Similarly, all diagonal elements of $\mathbf{\Sigma}_0^{-\frac{1}{2}}$ are

$$\left[\frac{1}{M}\frac{1}{\sqrt{1+(M-1)\rho}} + \left(1-\frac{1}{M}\right)\frac{1}{\sqrt{1-\rho}}\right]\frac{1}{\sigma},$$

and all off-diagonal elements are

$$\frac{1}{M}\left[\frac{1}{\sqrt{1+(M-1)\rho}} - \frac{1}{\sqrt{1-\rho}}\right]\frac{1}{\sigma}.$$

Hence we have (5) with β_1 given in (7) and β_2 given in (8). This also demonstrates the correct initial symmetry conjecture of matrices $\mathbf{\Gamma}$ and $\mathbf{\Lambda}$.

To obtain (11), notice that the insider's total expected profit conditional on v is $\frac{1}{2}\sigma_u(\boldsymbol{v}-\boldsymbol{p}_0)'\Sigma_0^{-\frac{1}{2}}(\boldsymbol{v}-\boldsymbol{p}_0)$. Thus the unconditional total expected profit

is

$$\begin{aligned} & E[\frac{1}{2}\sigma_u(\boldsymbol{v}-\boldsymbol{p}_0)'\Sigma_0^{-\frac{1}{2}}(\boldsymbol{v}-\boldsymbol{p}_0)] \\ = & \frac{1}{2}\sigma_u \mathrm{tr}\{\Sigma_0^{-\frac{1}{2}}E[(\boldsymbol{v}-\boldsymbol{p}_0)(\boldsymbol{v}-\boldsymbol{p}_0)']\} \\ = & \frac{1}{2}\sigma_u \mathrm{tr}\{\Sigma_0^{\frac{1}{2}}\}, \end{aligned}$$

or $\frac{1}{2}\sigma_u\sigma\left(\sqrt{1+(M-1)\rho} + (M-1)\sqrt{1-\rho}\right)$

A.2 Proof of Proposition 3.3

First, let $\rho = 0$ in the equations (7) - (11) from Proposition 3.1, we obtain the following:

$$\boldsymbol{\beta_{10} = \sigma_u/\sigma,\ \lambda_{10} = \sigma/2\sigma_u,\ \beta_{20} = \lambda_{20}=0,\ \ \text{and } E[\pi]_0 = M\sigma\sigma_u/2.}$$

Next, we take the first order derivatives of $(\beta_1,\beta_2,\lambda_1,\lambda_2,E[\pi])$ with respect to ρ to examine how they change with the correlation.

From (7), we have

$$\frac{\partial\beta_1}{\partial\rho} = \frac{M-1}{2M}\left\{-\frac{1}{[1+(M-1)\rho]^{\frac{3}{2}}} + \frac{1}{(1-\rho)^{\frac{3}{2}}}\right\}\frac{\sigma_u}{\sigma}.$$

Obviously, it is positive if $\rho > 0$ and negative if $-1/(M-1) < \rho < 0$. Hence, β_1 achieves the minimum at $\rho = 0$, or $0 < \beta_{10} < \beta_1$.

From (8), it is easy to see $\beta_2 < 0$ if $\rho > 0$ and $\beta_2 > 0 \text{ if } -\frac{1}{M-1} < \rho < 0$.
In addition, we have

$$\frac{\partial\beta_2}{\partial\rho} = \frac{1}{2M}\left\{-\frac{M-1}{[1+(M-1)\rho]^{\frac{3}{2}}} - \frac{1}{(1-\rho)^{\frac{3}{2}}}\right\}\frac{\sigma_u}{\sigma},$$

which is negative regardless of the sign of ρ.

From (9), it is easy to see $\lambda_1 > 0$ regardless of the sign of ρ. In addition, we have

$$\frac{\partial\lambda_1}{\partial\rho} = \frac{M-1}{2M}\left[\frac{1}{\sqrt{1+(M-1)\rho}} - \frac{1}{\sqrt{1-\rho}}\right]\frac{\sigma}{2\sigma_u},$$

which is negative if $\rho > 0$ and positive if $-\frac{1}{M-1} < \rho < 0$. Hence, λ_1 achieves the maximum at $\rho = 0$, or $0 < \lambda_1 < \lambda_{10}$.

From (10), it is easy to see $\lambda_2 > 0$ if $\rho > 0$ and $\lambda_2 < 0$ if $\rho < 0$. In addition, we have

$$\frac{\partial\lambda_2}{\partial\rho} = \frac{1}{2M}\left[\frac{M-1}{\sqrt{1+(M-1)\rho}} + \frac{1}{\sqrt{1-\rho}}\right]\frac{\sigma}{2\sigma_u},$$

which is positive regardless of the sign of ρ.

Finally, from (11), we have

$$\frac{\partial E[\pi]}{\partial\rho} = \frac{M-1}{4}\sigma_u\sigma\left[\frac{1}{\sqrt{1+(M-1)\rho}} - \frac{1}{\sqrt{1-\rho}}\right],$$

which is negative if $\rho > 0$ and positive if $-\frac{1}{M-1} < \rho < 0$. Hence, $E[\pi]$ achieves the maximum at $\rho = 0$, or $E[\pi] < E[\pi]_0$. Q.E.D.

Notes

[1]Some examples are Holden & Subrahmanyam (1992), Foster & Viswanthan (1994), Foster & Viswanthan (1996), Huddart et al. (2001), Caldentey & Stacchetti (2010), and Colla & Mele (2010).

[2]See Caballe & Krishnan (1994), for instance.

[3]As shown later, even a two-asset model is still unable to address certain issues inherent in multi-asset settings.

[4]Vitale (2012) examines the symmetry of the price impact of order flows across multiasset markets. A closely related literature is about multimarket trading such as Chowdhry & Nanda (1991) and Baruch et al. (2007), etc., where a single security is traded across multiple markets or within different exchanges. This is different from the multi-asset trading where several different securities are traded in a single market.

[5]In a conventional rational expectations equilibrium model, traders are usually assumed to know pricing rules but not actual prices in equilibrium.

[6] It is also called equicorrelation matrix, see Abadir & Magnus (2005). This type of matrices carry some excellent operation properties. Similar matrix has been used in the literature. For instance, Foster & Viswanthan (1996) used it to model the information structure of signals observed by differentially informed traders.

[7]At the extreme case when $\rho = -0.5$, the portfolio is negatively perfectly correlated with the third asset.

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